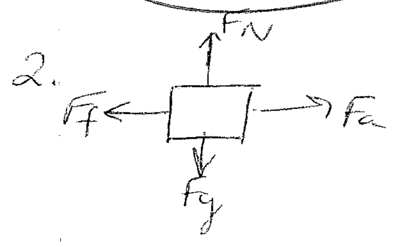
Dynamics Worksheets Phys 11H/12

ANSWERS KEY ①

2.



$\Sigma F_x = ma = F_a - F_f$

$\Sigma F_y = 0 = F_N - F_g$

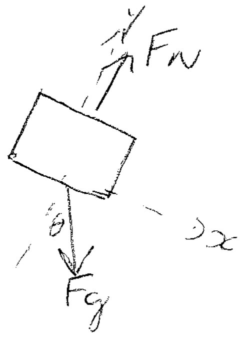
$\therefore F_N = F_g = mg$

$F_f = \mu mg$

$\therefore ma = F_a - \mu mg$

$\therefore \boxed{a = \frac{F_a - \mu mg}{m}}$

oops! (out of order) → 5. (a)



$\Sigma F_y = 0 = F_N - F_g\cos\theta$

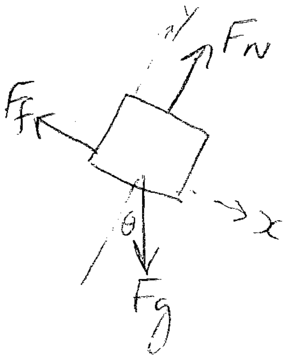
$\mu = 0$

$\Sigma F_x = F_g\sin\theta = ma$

$\cancel{m}g\sin\theta = \cancel{m}a$

$\therefore \boxed{a = g\sin\theta}$

(b)



$\Sigma F_y = 0 = F_N - F_g\cos\theta$

$\therefore F_N = mg\cos\theta$

$\therefore F_f = \mu F_N = \mu mg\cos\theta$

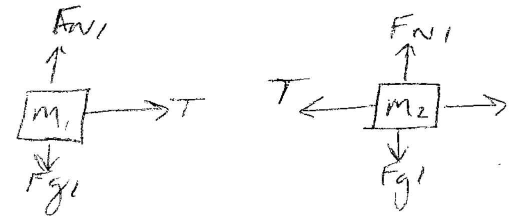
$\Sigma F_x = ma = F_g\sin\theta - F_f$

$\cancel{m}a = \cancel{m}g\sin\theta - \mu \cancel{m}g\cos\theta$

$\therefore \boxed{a = g(\sin\theta - \mu\cos\theta)}$

(oops! out of order)

3.



$\Sigma F_y = 0$

① $\Sigma F_{1x} = m_1 a = T$ $(\mu = 0)$

② $\Sigma F_{2x} = m_2 a = F_p - T$

substitute ① into ②

$a_1 = a_2 = a$

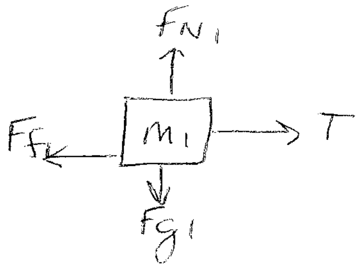
$m_2 a = F_p ~~\#~~ - m_1 a$

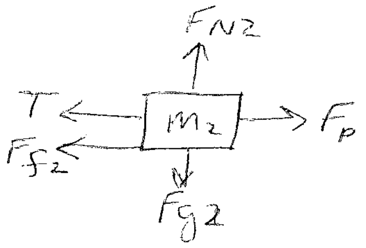
$\therefore m_1 a + m_2 a = F_p$

$a(m_1 + m_2) = F_p$

$\therefore \boxed{a = \frac{F_p}{m_1 + m_2}}$

4.





$a_1 = a_2 = a$

$\Sigma F_{y1} = 0 = F_{N1} - F_{g1}$

$\therefore F_{N1} = F_{g1}$

$F_{f1} = \mu F_{N1}$

$\therefore F_{f1} = \mu m_1 g$

$\Sigma F_{1x} = m_1 a = T - F_f$

$m_1 a = T - \mu m_1 g$

① $\therefore T = m_1 a + \mu m_1 g$

$\Sigma F_{y2} = 0 = F_{N2} - F_{g2}$

$\therefore F_{N2} = F_{g2}$

$F_{f2} = \mu F_{N2} = \mu m_2 g$

$\Sigma F_{2x} = m_2 a = F_p - T - F_{f2}$

② $m_2 a = F_p - T - \mu m_2 g$

substitute ① into ②

$$m_2 a = F_p - (m_1 a + \mu m_1 g) - \mu m_2 g$$

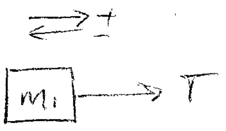
$$m_2 a + m_1 a = F_p - \mu m_1 g - \mu m_2 g$$

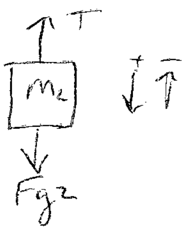
$$a(m_1 + m_2) = F_p - \mu g(m_1 + m_2)$$

$$\boxed{a = \frac{F_p - \mu g(m_1 + m_2)}{(m_1 + m_2)}}$$

(#5 answer is on pg 1)

6.





$a_1 = a_2 = a$

$\Sigma F_1 = m_1 a = T$

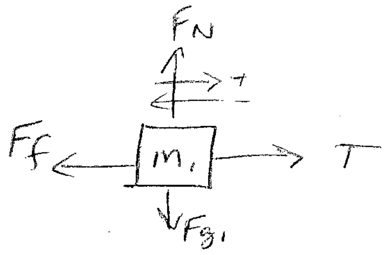
$\Sigma F_2 = \underline{m_2 a = F_{g2} - T}$ add

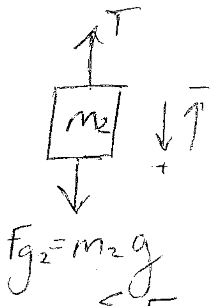
$m_1 a + m_2 a = F_{g2}$

$a(m_1 + m_2) = m_2 g$

$$\boxed{a = \frac{m_2 g}{m_1 + m_2}}$$

7.





$a_1 = a_2 = a$

$\Sigma F_{y1} = 0 = F_N - F_{g1}$

$F_N = \cancel{\mu} F_{g1} = m_1 g$

$F_f = \mu F_N = \mu m_1 g$

$\Sigma F_{x1} = m_1 a = T - F_f$

① $m_1 a = T - \mu m_1 g$

① $T = m_1 a + \mu m_1 g$

$\Sigma F_{y2} = m_2 a = F_{g2} - T$

② $m_2 a = m_2 g - T$

substitute ① into ②

$m_2 a = m_2 g - (m_1 a + \mu m_1 g)$

$m_2 a + m_1 a = m_2 g - \mu m_1 g$

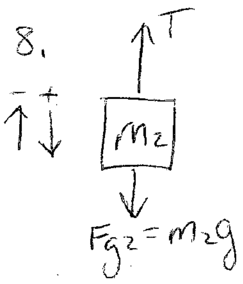
$a(m_1 + m_2) = g(m_2 - \mu m_1)$

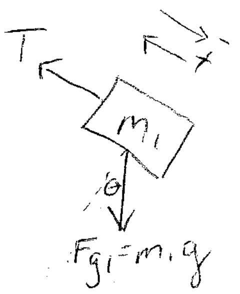
$$\boxed{a = \frac{g(m_2 - \mu m_1)}{m_1 + m_2}}$$

if $a \geq 0$ $\mu \cancel{\leq} \frac{m_2}{m_1}$

$a_1 = a_2 = a$

8.





$\Sigma F_2 = m_2 a = F_{g2} - T$

① $m_2 a = m_2 g - T$

$\Sigma F_1 = m_1 a = T - F_{g1} \sin\theta$

② $m_1 a = T - m_1 g \sin\theta$

~~ΣF_1~~ add ① and ②

① $m_2 a = m_2 g - T$

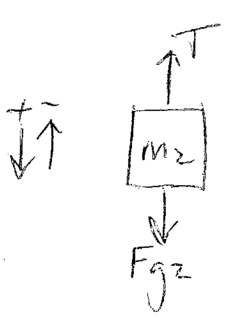
② $m_1 a = T - m_1 g \sin\theta$

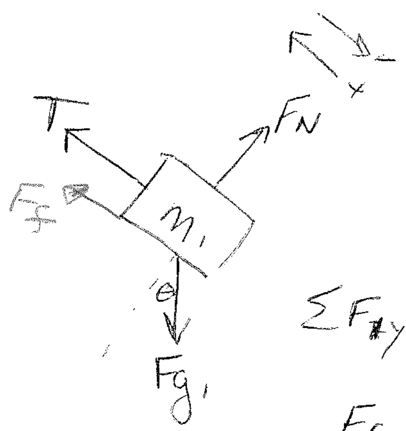
$m_1 a + m_2 a = m_2 g - m_1 g \sin\theta$

$a(m_1 + m_2) = g(m_2 - m_1 \sin\theta)$

$$\boxed{a = \frac{g(m_2 - m_1 \sin\theta)}{m_1 + m_2}}$$

9.





$a_1 = a_2 = a$

$\Sigma F_{\cancel{x}y} = 0 = F_N - F_{g1}\cos\theta$

$F_N = m_1 g \cos\theta$

$F_f = \mu F_N = \mu m_1 g \cos\theta$

$\Sigma F_{\cancel{y}x} = m_1 a = T - F_{g1}\sin\theta - F_f$

② $m_1 a = T - m_1 g \sin\theta + \mu m_1 g \cos\theta$

$\Sigma F_2 = m_2 a = F_{g2} - T$

① $m_2 a = m_2 g - T$

$T = m_2 g - m_2 a$

Substitute ① into ②

$m_1 a = m_2 g - m_2 a - m_1 g \sin\theta + \mu m_1 g \cos\theta$

$m_1 a + m_2 a = m_2 g - m_1 g \sin\theta + \mu m_1 g \cos\theta$

$a(m_1 + m_2) = g(m_2 - m_1 \sin\theta + \mu m_1 \cos\theta)$

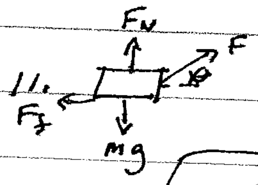
$$\boxed{a = \frac{g(m_2 - m_1 \sin\theta + \mu m_1 \cos\theta)}{m_1 + m_2}}$$

if $m_2 > m_1 \sin\theta$ system will tend to want to slide to left (and F_f will be to the right)

if $m_2 < m_1 \sin\theta$ system will tend to want to slide to right (and F_f will be to the left)

10. $$\boxed{a = \frac{g(m_2 \sin\phi - m_1 \sin\theta)}{m_1 + m_2}}$$

11.



$\Sigma F_y = 0 = F_N + F\sin\theta - mg$

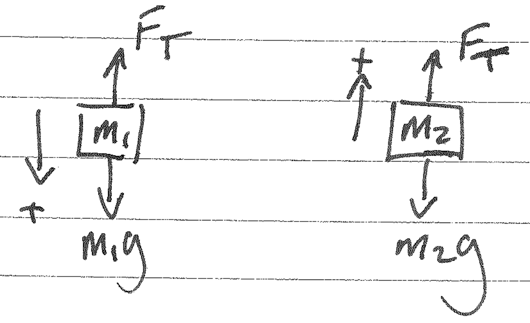
$F_N = mg - F\sin\theta$

$$\boxed{a = \frac{F\cos\theta - \mu(mg - F\sin\theta)}{m}}$$

②

pg3

1. (a)



(b) $a = \frac{g(m_1 - m_2)}{m_1 + m_2} = \frac{(9.8)(5.6 - 3.8)}{5.6 + 3.8} = 1.8765$

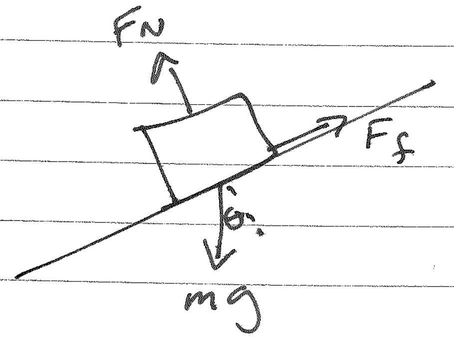
$\boxed{a = 1.9 m/s^2}$

(c) $m_1 a = m_1 g - F_T \quad \therefore F_T = m_1(g - a)$

$= (5.6)(9.8 - 1.876)$

$\boxed{F_T = 44 N}$

2.



$\Sigma F = \boxed{\cancel{m}a = \cancel{m}g\sin\theta - \mu\cancel{m}g\cos\theta}$

$\vec{a} = 0$

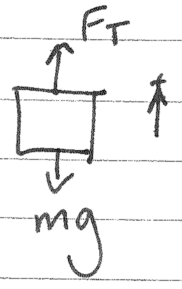
$\therefore \sin\theta = \cancel{\mu}\cos\theta$

$\therefore \theta = \text{tan}^{-1}[\cancel{\mu}] = \tan^{-1} 0.18$

(a) + (b) $\boxed{\theta = 10.2°}$

pg4

(a)

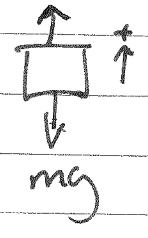


$ma = F_T - mg$

$a = \frac{(2.8 \times 10^4) - (1555)(9.8)}{1555}$

$\boxed{a = 8.2 m/s^2}$

(b)

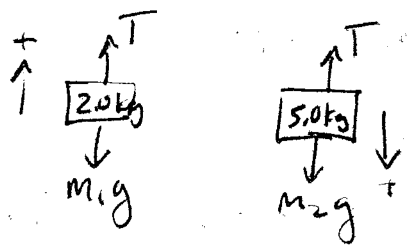


$ma = F_N - mg$

$F_N = m(a + g) = (55)(-1.5 + 9.8)$

$\boxed{F_N = 4.6 \times 10^2 N}$

①



$$\Sigma F_1 = m_1 a = T - m_1 g$$
$$\Sigma F_2 = m_2 a = m_2 g - T$$

①

$$m_1 a + m_2 a = m_2 g - m_1 g$$
$$a = \frac{g(m_2 - m_1)}{m_1 + m_2}$$

3

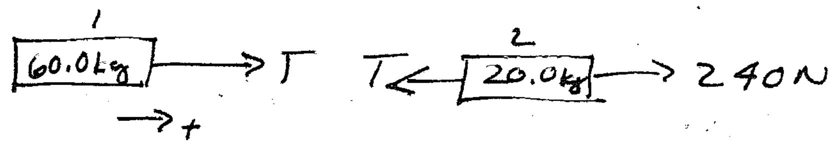
$$a = \frac{(9.8 m/s^2)(5.0 kg - 2.0 kg)}{2.0 kg + 5.0 kg}$$

①

$$\boxed{a = 4.2 m/s^2 \text{ up}}$$

①

②



$$\Sigma F_1 = m_1 a = T$$

①

$$T = (60.0 kg)(3.00 m/s^2)$$

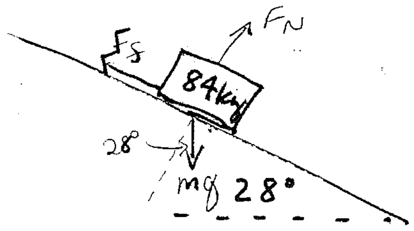
Ⓒ $\boxed{T = 180 N}$

③ $F_R = \sqrt{4^2 + 3^2} = \sqrt{16 + 9}$

①

$\boxed{B}$ $F_R = 5.0 N$

④



$$F_f = \mu F_N$$
$$F_N = mg \cos 28°$$
$$F_f = \mu mg \cos 28°$$
$$\mu = \frac{180 N}{(84 kg)(9.8 N/kg) \cos 28°} = \boxed{0.25}$$

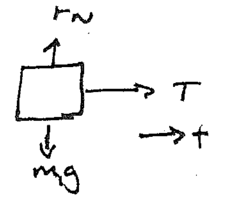
⑤ ~~F=~~ $\Delta p = F\Delta t$

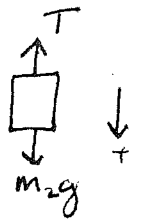
$$F = \frac{\Delta p}{\Delta t} = \frac{\Delta(mv)}{\Delta t} = \frac{(0.020 kg)(400 m/s)}{0.12 s} = 66.7 N$$

$$f = \frac{500}{60 s} =$$
$$\therefore T = \frac{60 s}{500} = 0.12 s$$

⑤

① (a)





⑥

$\Sigma F_1 = m_1 a = T$

$\Sigma F_2 = m_2 a = M_2 g - T$

$(m_1 + m_2) a = m_2 g$

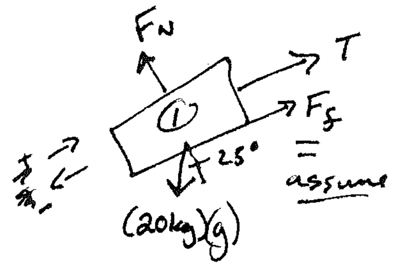
$a = \frac{m_2 g}{m_1 + m_2} = \frac{(12\,kg)(9.8\,N/kg)}{15\,kg + 12\,kg}$

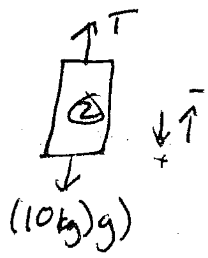
$\boxed{a = 4.4\,m/s^2}$

(b) $T = m_1 a = 15\,kg\,(4.4\,m/s^2)$

$\boxed{T = 65\,N}$

②





$\Sigma F_1 = m_1 a = T - (20\,kg) g \sin 25° + F_f$

$F_f = \mu m_1 g \cos 25°$

$\Sigma F_2 = m_2 a = (10\,kg) g - T$

$\therefore T = (10\,kg) g - m_2 a$

$m_1 \overset{0}{a} = (10\,kg) g - m_2 \overset{0}{a} - (20\,kg) g \sin 25° + \overset{F_f}{\cancel{\mu m_1 g \cos 25°}}$

at rest $\underline{a = 0}$

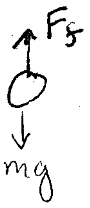
$F_f = (20\,kg) g \sin 25° - (10\,kg) g$

$\boxed{F_f = -15\,N}$ (negative ∴ direction in diagram opposite to true dir.)

∴ direction is <u>down</u> the slope

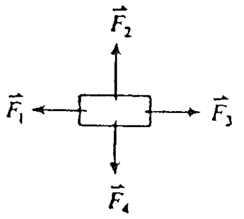
③ $\boxed{C}$

④



forces ballanced

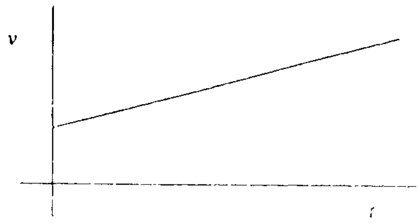
1. **The free body diagram shown below represents a crate being dragged towards the left over a rough surface.**



Which of the vectors represent the normal force and the friction force acting on the crate?

	NORMAL FORCE	FRICTION FORCE
A.	$\vec{F}_1$	$\vec{F}_2$
[✓ B.]	$\vec{F}_2$	$\vec{F}_3$
C.	$\vec{F}_3$	$\vec{F}_4$
D.	$\vec{F}_4$	$\vec{F}_1$

2. **The graph shown below displays velocity v versus time t for a moving object.**



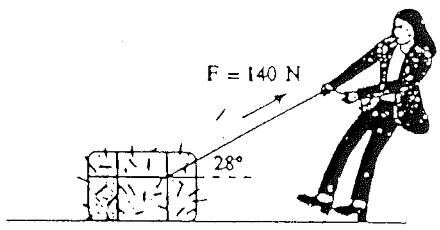
The slope of this graph represents the object's

A. mass.
B. momentum.
[✓ C.] acceleration.
D. displacement.

3. The gravitational field strength on planet X is 5.0 N/kg. An astronaut of mass 60 kg leaves Earth to visit planet X. What will her mass and weight be when she is on the surface of planet X?

	MASS	WEIGHT
[✓ A.]	60 kg	300 N
B.	60 kg	590 N
C.	~~120 kg~~	300 N
D.	~~120 kg~~	590 N

4. **A girl applies a 140 N force to a 35 kg bale of hay at an angle of 28° above horizontal. The friction force acting on the bale is 55 N. What will be the horizontal acceleration of the bale?**

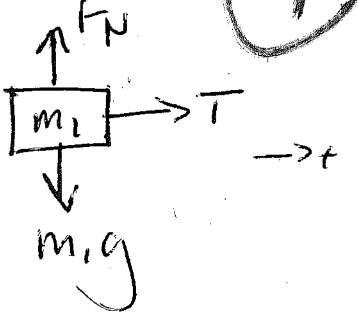


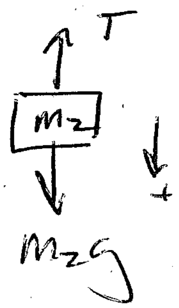
$$a = \frac{140\cos 28° - 55}{35}$$

A. 0.31 m/s^2
[✓ B.] 2.0 m/s^2
C. 2.4 m/s^2
D. 2.6 m/s^2

#5 ⑨

1. (a)





(b) $\Sigma F_1 = m_1 a = T \Rightarrow \therefore a = \frac{T}{m_1}$

$\Sigma F_2 = m_2 a = m_2 g - T$

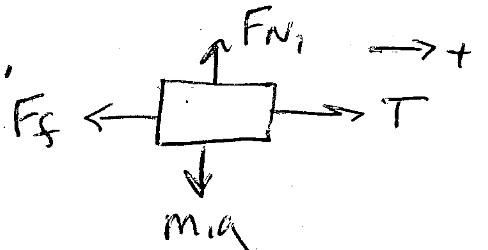
$$m_2\left(\frac{T}{m_1}\right) = m_2 g - T$$

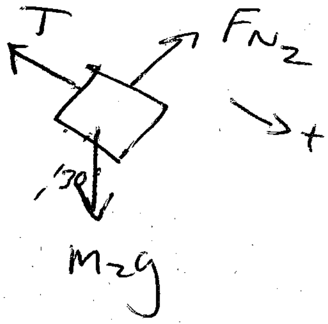
$$m_2 T = m_1 (m_2 g - T)$$

$$\therefore m_1 = \frac{m_2 T}{m_2 g - T} = \frac{(6.1)(43)}{(6.1)(9.8) - 43}$$

$$\boxed{m_1 = 16\,kg}$$

2.





① $\Sigma F_1 = m_1 a = T - \mu m_1 g$

② $\Sigma F_2 = m_2 a = m_2 g \sin 30° - T$

from ① $a = \frac{T - \mu m_1 g}{m_1}$

into ② $m_2\left(\frac{T - \mu m_1 g}{m_1}\right) = m_2 g \sin 30° - T$

$$\frac{m_2 T}{m_1} - \frac{\mu m_1 \overset{m_2}{\cancel{m_2}} g}{\cancel{m_1}} = m_2 g \sin 30° - T$$

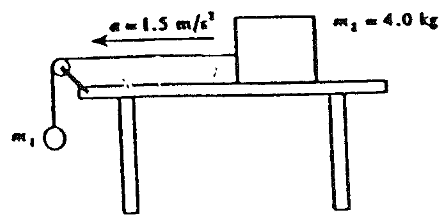
$$\left(\frac{m_2}{m_1} + 1\right) T = m_2 g \sin 30° + \mu m_2 g$$

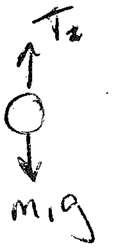
$$T = \frac{(4)(9.8)\sin 30° + (0.2)(4)(9.8)}{\left(\frac{4}{6} + 1\right)}$$

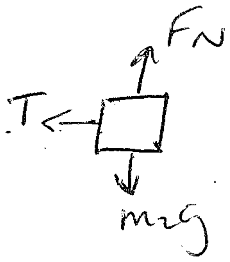
$$\boxed{T = 16N}$$

10

3. The 4.0 kg block shown accelerates across a frictionless horizontal table at 1.5 m/s².







Find the mass of object m_1.

A. 0.61 kg
[✓ B. 0.72 kg]
C. 6.0 kg
D. 26 kg

$$a = \frac{m_1 g}{m_1 + m_2}$$

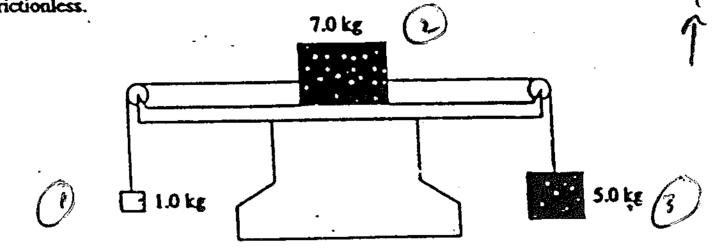
$$am_1 + am_2 = m_1 g$$

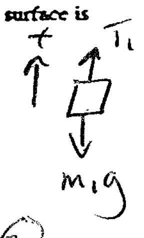
$$am_2 = m_1 g - m_1 a$$

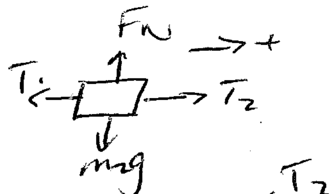
$$m_1 = \frac{am_2}{g-a} = \frac{(1.5)(4)}{9.8 - 1.5}$$

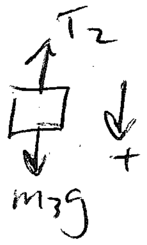
$$= 0.72$$

4. Three blocks have masses 1.0 kg, 7.0 kg and 5.0 kg as shown. The horizontal surface is frictionless.









What is the magnitude of the acceleration of the system?

[✓ A. 3.0 m/s²]
B. 3.8 m/s²
C. 6.5 m/s²
D. 7.8 m/s²

$$m_1 a = T_1 - m_1 g$$

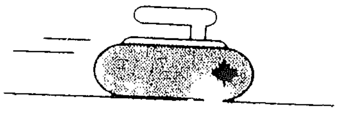
$$m_2 a = T_2 - T_1$$

$$m_3 a = m_3 g - T_2 \quad \underline{\text{add}}$$

$$m_1 a + m_2 a + m_3 a = m_3 g - m_1 g$$

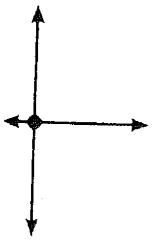
$$a = \frac{g(m_3 - m_1)}{m_1 + m_2 + m_3}$$

1. A curling rock is travelling to the right across the ice as shown in the diagram.



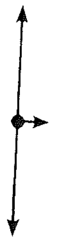
Which of the following best represents the forces acting on the curling rock?

A.

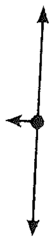


B.

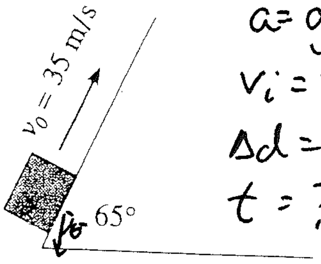
C.



[✓ D.]



2. An object is fired up a frictionless ramp as shown in the diagram.



$a = g\sin 65°$

$v_i = 35$

$\Delta d = 0$

$t = ?$

$\overset{0}{\cancel{\Delta d}} = \frac{1}{2}a\cancel{t}^{2} + v_i\cancel{t}$

$t = \frac{2V_i}{a}$

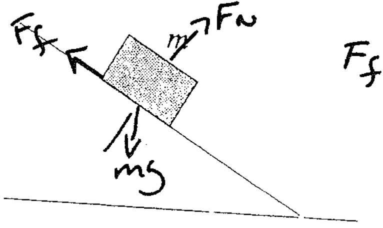
$= \frac{2(35)}{(9.8)\sin 65°}$

$=$ ______

If the initial velocity is 35 m/s, how long does the object take to return to the starting point?

A. 3.6 s
B. 3.9 s
C. 7.9 s
D. 17 s

3. A block of mass *m* remains at rest on an incline as shown in the diagram.

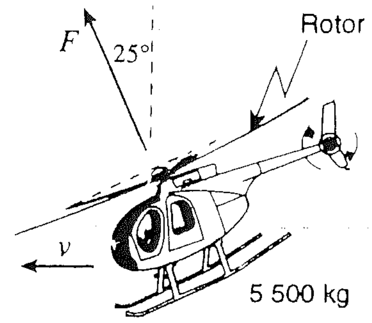


$F_f = \mu mg\cos\theta = mg\sin\theta$

The force acting up the ramp on this block is

A. 0.
B. mg.
[✓ C.] less than mg.
D. more than mg.

4. A 5 500 kg helicopter is travelling at constant speed in level flight.



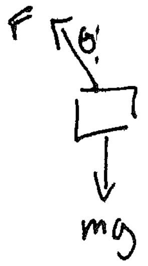
What is the force F provided by the rotor?

A. 4.9×10^{4} N

B. 5.4×10^{4} N

C. 5.9×10^{4} N

D. 1.2×10^{5} N



$F\cos25^{\circ} = mg$

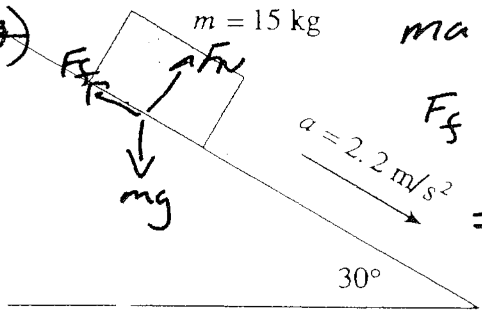
$F = \frac{(5500)(9.8)}{\cos25^{\circ}}$

$=$

5. A 15 kg block has a constant acceleration of 2.2 m/s^2 down a 30° incline.

$a = g(~~\sin\theta - \mu\cos\theta~~)$

~~2.2 = 9.8~~



$ma = mg\sin\theta - F_f$

$F_f = m(g\sin\theta - a)$

$= (15)(9.8\sin30^{\circ} - 2.2)$

$=$

What is the magnitude of the friction force on the block?

A. 33 N
B. 41 N
C. 74 N
D. 130 N

6. Force F gives mass m_1 an acceleration of 4.0 m/s^2. The same force F gives mass m_2 an acceleration of 2.0 m/s^2. What acceleration would force F give to the two masses m_1 and m_2 if they were glued together?

$F = m_1 \times 4$ $\quad\therefore m_1 = \frac{F}{4}$ $\quad\left(\frac{F}{4} + \frac{2F}{4}\right)a = F$

$F = m_2 \times 2$ $\quad m_2 = \frac{F}{2}$ $\quad a = \frac{4F}{3F} = 1.25$

A. 1.0 m/s^2

[✓ B.] 1.3 m/s^2

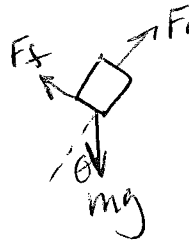
C. 3.0 m/s^2

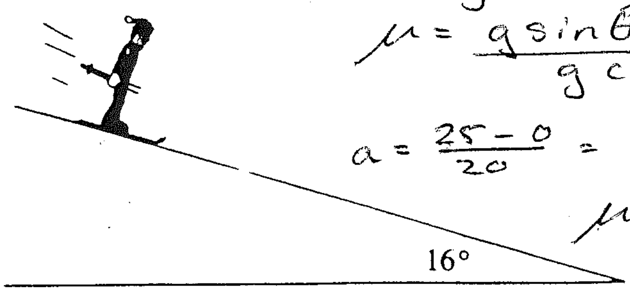
D. 6.0 m/s^2

pg 13

Dyn. WS. #4

1. A 75 kg Olympic skier takes 20 s to reach a speed of 25 m/s from rest while descending a uniform 16° slope.





$\cancel{m}a = \cancel{m}g\sin\theta - \mu\cancel{m}g\cos\theta$

$\mu = \dfrac{g\sin\theta - a}{g\cos\theta}$

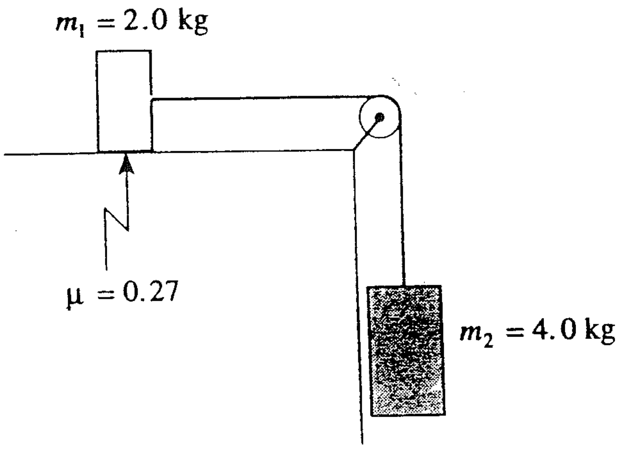
$a = \dfrac{25-0}{20} = 1.25$

$\mu = \dfrac{9.8\sin 16 - 1.25}{9.8\cos 16}$

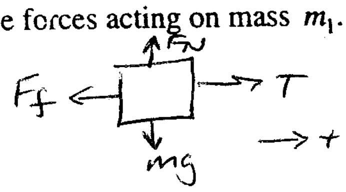
$\boxed{\mu = 0.15}$

What is the coefficient of friction between the skis and the slope surface? **(7 marks)**

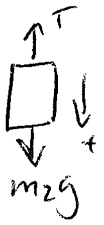
2. Two masses are connected by a light string over a frictionless massless pulley. There is a coefficient of friction of 0.27 between mass m_1 and the horizontal surface.



a) Draw and label a free body diagram showing the forces acting on mass m_1. **(2 marks)**



b) What is the acceleration of mass m_2? **(5 marks)**



$m_2a = m_2g - T$

$m_1a = T - \mu m_1g$

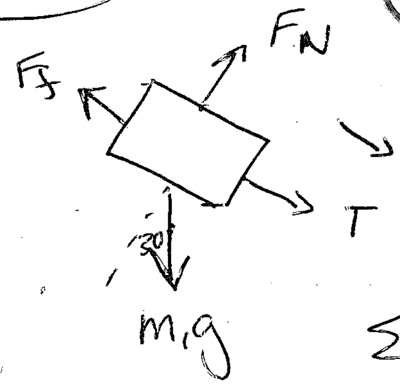
$a(m_1+m_2) = g(m_2 - \mu m_1)$

$a = \dfrac{(9.8)(4 - 0.27 \times 2)}{6}$

$\boxed{a = 5.7\,m/s^2}$

S #4

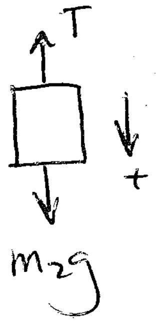
3.



$F_f = \mu m_1 g\cos\theta$

$\Sigma F_1 = m_1a = T + m_1g\sin\theta - \mu m_1 g\cos\theta$

$\Sigma F_2 = m_2a = m_2g - T$



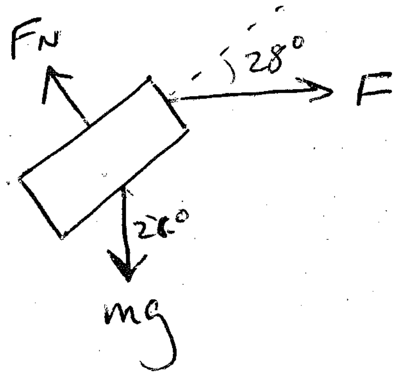
$(m_1+m_2)a = m_2g + m_1g\sin\theta - \mu m_1g\cos\theta$

$$a = g\frac{(m_2 + m_1\sin\theta - \mu m_1\cos\theta)}{m_1+m_2}$$

$$a = \frac{9.8(3 + 8\sin 30 - 0.26 \times 8 \times \cos 30)}{11}$$

$$\boxed{a = 4.6 m/s^2}$$

4. (a)



(b) $\Sigma F_x = m\overset{0}{a} = F\cos 28° - mg\sin 28°$

$\therefore F = \frac{mg\sin 28°}{\cos 28°}$

$$\boxed{F = 65N}$$

(c) $\Sigma F_y = 0 = F_N - mg\cos 28° - F\sin 28°$

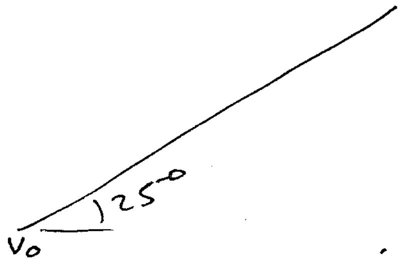
$F_N = (12.5)(9.8)\cos 28° + (65.1)\sin 28°$

$$\boxed{F_N = 1.4 \times 10^2 N}$$

15

Calvin + Hobbes ws5

1) a)



$V_o = 12.5 m/s$
$a = -g\sin\theta$
$V_f = ?$
$\Delta d = 17.0 m$

$V_f = \sqrt{2a\Delta d + V_o^2}$

$V_f = \sqrt{2(-9.8)\sin 25(17) + 12.5^2}$

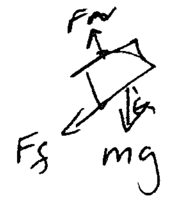
$\boxed{V_f = 3.9 m/s}$

∴ still moving!

b) $V_o = 12.5 m/s$
$V_f = 0$
$\Delta d = 17.0 m$
$a = ?$

$a = \frac{V_f^2 - V_o^2}{2\Delta d} = \frac{0 - (12.5)^2}{2(17)}$

$a = -4.596 m/s^2$



$\cancel{m}a_x = -\cancel{m}g\sin\theta - \mu \cancel{m}g\cos\theta$

$\mu = \frac{-a - g\sin\theta}{g\cos\theta} = -\frac{(-4.596) - (9.8)\sin 25}{(9.8)\cos 25}$

$\boxed{\mu = 0.051}$

c) $a = -4.596 m/s^2$
$V_o = 12.5 m/s$
$\Delta d = ?$
$\Delta t =$

$\Delta d = \frac{1}{2}at^2 + V_o t$

(i) $\Delta d = \frac{1}{2}(-4.596)(0.5)^2 + (12.5)(0.5)$

$\boxed{\Delta d = 5.7 m}$

at top $V_f = 0$
$\Delta d = 17$
$a = -4.596$
$V_o = 12.5$
$\Delta t = ?$

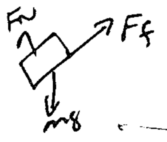
$t = \frac{V_f - V_o}{a} = 2.72 s$

(ii) $\Delta d = \frac{1}{2}(-4.596)(1.5)^2 + (12.5)(1.5)$
$= 13.5795$

$\boxed{\Delta d = 13.6 m}$

(iii) $\Delta d = \frac{1}{2}(-4.596)(2.5)^2 + (12.5)(2.5)$

$\boxed{\Delta d = 16.9 m}$



change dir.

$a = -g\sin\theta + \mu g\cos\theta$
$= -3.689$

from top.
$V_o = 0$
$\Delta t = t - 2.72 s$

(iv) $\Delta d = \frac{1}{2}(-3.689)(3 - 2.72)^2 + 0 = 0.1446$
$\Delta d = 17 - 0.1446$

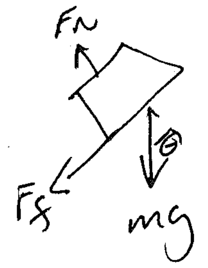
$\boxed{\Delta d = 16.9 m}$

(v) $\Delta d = \frac{1}{2}(3.689)(5 - 2.72)^2$

$\Delta d = 17 -$

$\boxed{\Delta d = 7.4 m}$

2. on ramp



$ma = -mg\sin\theta - \mu mg\cos\theta$

$a = -g(\sin\theta + \mu\cos\theta)$

$V_o = 11.0\,m/s$

$V_f = ?$

$\Delta d = 10.0\,m$

$V_f^2 = 2a\Delta d + V_o^2$

$V_f = \sqrt{2(-9.8)(\sin 27 + 0.07\cos 27)(10) + 11^2}$

$= 4.45\,m/S$

as projectile

x	y
$V_x = 4.45\cos 27°$	$V_{oy} = 4.45\sin 27°$
$\Delta d_x = x$	$a_y = -9.8$
$t =$	$\Delta d_y = -15$
	$\Delta t = ?$

$\Delta d_y = \frac{1}{2}a_y t^2 + V_{oy} t$

$0 = \frac{1}{2}a_y t^2 + V_{oy} t - \Delta d_y$

$0 = \frac{1}{2}(-9.8)t^2 + 4.45\sin 27° t - (-15)$

$0 = -4.9t^2 + 2.02t + 15$

$t = \frac{-2.02 \pm \sqrt{2.02^2 - 4(-4.9)(15)}}{2(-4.9)}$

$= \frac{-2.02 \pm 17.27}{-9.8} = 1.968$

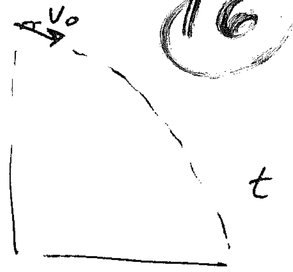
$\Delta d_x = V_x t$

$= (4.45\cos 27°)(1.968)$

$\boxed{\Delta d_x = 7.8m}$

as projectile

(3.)



16

x

$\Delta d_x = 7.4$

$V_x = V_o \cos\theta$

$V_x = \frac{d_x}{t}$

$t = \frac{d_x}{V_x}$

$= \frac{7.4}{V_o \cos\theta}$

y

$\Delta d_y = 8.0\,m$

$a_y = -9.8$

$V_{oy} = V_o \sin\theta$

$\Delta t = ?$

$$\Delta d_y = \tfrac{1}{2} a_y t^2 + V_{oy} t$$

$$8 = \tfrac{1}{2}(9.8)\left(\frac{7.4}{V_o \cos 22}\right)^2 + (\cancel{V_o}\sin 22)\left(\frac{7.4}{\cancel{V_o}\cos\theta}\right)$$

$$5.01 = \frac{312.12}{V_o^2}$$

$$V_o^2 = \frac{312.12}{5.01} \quad \therefore V_o = 7.893\,m/s$$

---

on roof

$V_o = 0$

$V_f = 7.893$

$a = 9.8(\sin 22° - \mu \cos 22°)$

$\Delta d = 12\,m$

$$V_f^2 = 2a\Delta d + V_o^2$$

$$7.893^2 = 2(9.8)(\sin 22° - \mu\cos 22°)(12) + 0$$

$$(12)2(9.8)\mu\cos 22° = (12)2(9.8)\sin 22° - 7.893^2$$

$$\boxed{\mu = 0.12}$$