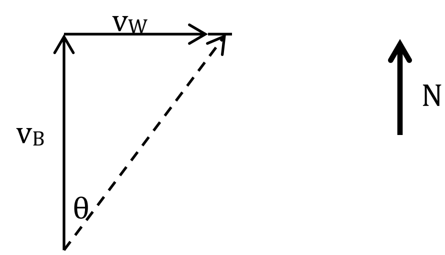
Rivers and Airplanes Worksheet Answer Key

Part 2 – Analytical Methods



1.

(a) $v_R = \sqrt{5^2 + 3^2} = 5.8$ m/s $\theta = \tan^{-1}[3/5] = 31°$

Therefore: **v_R = 5.8 m/s [31° E of N]**

(b) The time to get across the river depends upon the component of velocity that is across the river (North) rather than the component that points up or down the river (East or West).

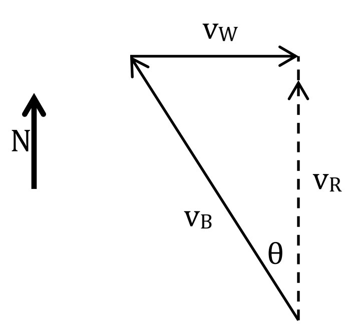
$v = d/t$ therefore $t = d/v$

So, $t = \frac{d_N}{v_N} = \frac{300.0\text{m}}{5.0\text{m/s}} = 60\text{s} =$ **6.0×10^{1} s**

(a) The distance travelled downstream depends upon the downstream component of velocity (East) rather than the component of velocity that points across the river (North).

$d_E = v_E \times t = (3.0\text{m/s})\times(60\text{s}) =$ **1.8×10^{2} m**

2.



(a) $\theta = \sin^{-1}[3/5] = 37°$

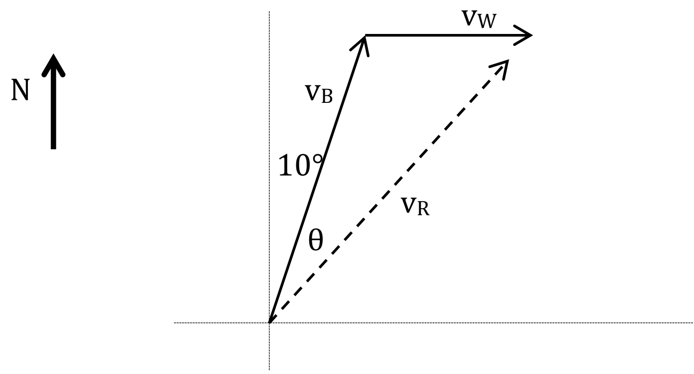
therefore, the **heading is 37° W of N**

(b) $v_R = \sqrt{5^2 - 3^2} =$ 4.0 m/s

therefore **v_R = 4.0 m/s [due N]**

(c) $t = \frac{d_N}{v_N} = \frac{300\text{m}}{4.0\text{m/s}} =$ **75 s**

3. (a) This problem can be solved using cosine law and sine law, or using the component method.



Method 1: Cosine Law and Sine Law

Cosine law: $v_R^2 = 5^2 + 3^2 - 2(5)(3)\cos 100°$ therefore: $v_R = 6.26$ m/s

Sine law: $\dfrac{\sin 100°}{6.26} = \dfrac{\sin\theta}{3.0}$ therefore: $\theta = 28°$

But, direction of motion of the boat = $\theta + 10.0° = 28° + 10.0° = 38°$

Therefore: <u>$v_R = 6.3$ m/s [38° E of N]</u>

Method 2: Component Method

x direction (East – West axis): $v_x = [(5.0\text{m/s})\times(\sin 10.0°)] + 3.0\text{m/s} = 3.8682$ m/s

y direction (North – South axis): $v_y = (5.0\text{m/s})\times(\cos 10.0°) = 4.924$ m/s

$v_R = \sqrt{v_x^2 + v_y^2} = 6.26$ m/s

$\theta = \tan^{-1}[v_x/v_y] = 38.15°$

Therefore: <u>$v_R = 6.3$ m/s [38° E of N]</u>

(b) $t = \dfrac{d_N}{v_N} = \dfrac{300\text{m}}{(5.0\text{m/s})(\cos 10.0°)} = 60.9\text{s} =$ **<u>61 s</u>**

(c) $d_E = v_E \times t = (6.26\text{m/s})\times(\sin 38°)\times(60.9\text{s}) =$ **<u>2.3×10^2 m</u>**

4. To get across the river in the shortest time the boat must head in the direction such that the longest possible component of the resultant velocity will be pointed toward the North (directly across the river). When the boat is headed North, the Northward component of velocity is 5.0 m/s. When the boat is headed East or West of North, the Northward component is less than 5.0 m/s. So, for the fastest crossing of the river: **<u>Heading = due North</u>**