Corrected Nov 2017 (With answers at end)

Kinematics in 2-D Booklet

(Practice Problems)

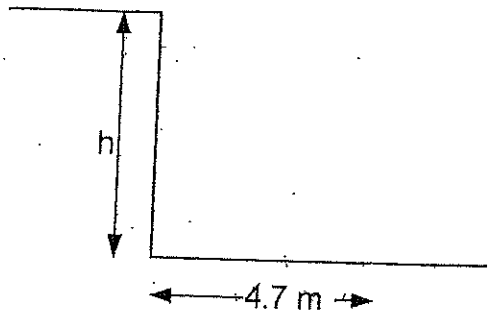
Posted on website

1. To reach its destination on time, an airplane must travel at 475 km/h relative to the ground in a direction 65.0° south of east. If the wind velocity is 45.0 km/h towards the east, what is the velocity of the plane relative to the air (speed and direction) ?

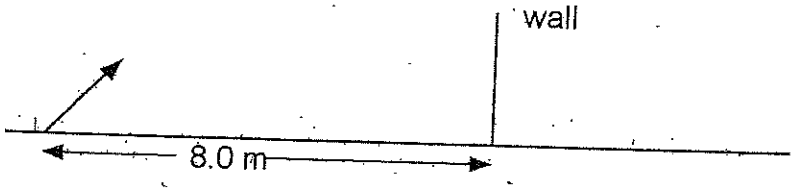
2. Sally the diver made a daring leap from the top of a cliff. She started her dive by running straight off the cliff (horizontally) with a speed of 2.50 m/s. She hit the water 4.7 m from the base of the cliff.

(a) How long was she in the air?

(b) How high is the cliff?



3. A water hose lying on the ground shoots a stream of water upward at an angle of 40.0° to the horizontal. The speed of the water is 20.0 m/s as it leaves the hose. How high up will it strike the wall which is 8.0 m away?



4. A man standing on the edge of a vertical cliff, 45.0 m high, throws a ball upward at an angle of 25.0° above the horizontal with a speed of 7.9 m/s.

(a) How long will it take for the ball to hit the ground?

(b) What is the velocity of the ball the instant before it hits the ground?

5. A helicopter was moving upwards at a constant rate of 5.0 m/s. When the helicopter was 250 m above the ground, the pilot threw her apple core horizontally out the window with a speed of 7.0 m/s (relative to the helicopter). How long was the apple core in the air?

6 An airplane whose airspeed is 210 km/h heads due west. A wind is blowing from north to south at 85.0 km/h.

(a) What is the **direction** that the plane actually travels?

(b) What is the **speed** of the plane with respect to the ground?

(c) If the pilot wants the plane to travel due west, in what direction should the plane head ?

7. A 2.80 kg projectile is launched at 24.0 m/s from level ground. The launch angle is 31.0 ° above the horizontal.

(a) What is the maximum height reached by the projectile?

(b) How fast will the projectile be travelling when it is at its maximum height?

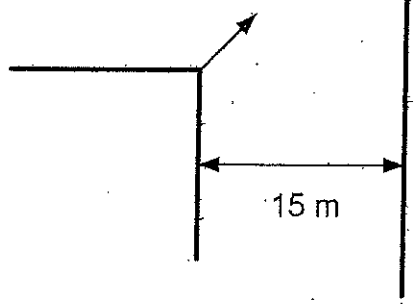
8. A rock is thrown horizontally off a cliff with a speed of 26.0 m/s. The cliff is 35.0 m above a lake.

(a) How long will it take for the rock to hit the lake?

(b) What is the rock's velocity at the moment of impact with the water?

(c) How far from the cliff will the rock land? (horizontal distance)

9. A ball is thrown from the top of one building toward a tall building 15 m away. The initial velocity of the ball is 7.0 m/s at 40.0° above the horizontal. How far above or below its original level will the ball strike the opposite wall?

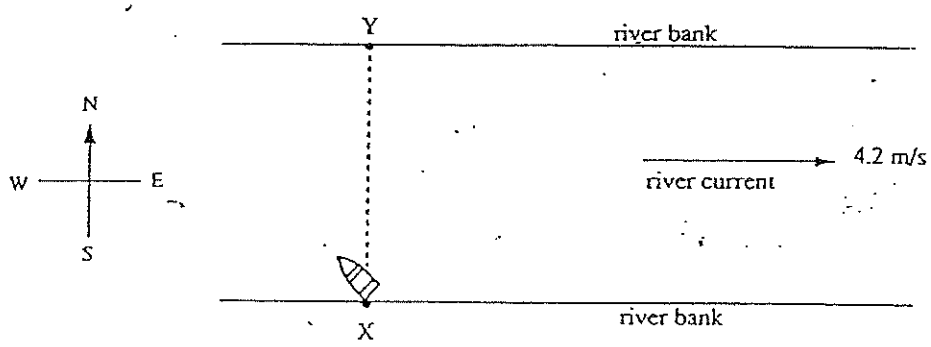


10. A helicopter was dropping towards the earth at a constant rate of 5.0 m/s. When the helicopter was 250 m above the ground, the pilot threw her apple core horizontally out the window with a speed of 7.0 m/s (relative to the helicopter). How long was the apple core in the air?

10. A rock is falling from a building. While the rock is falling, which of the following remains constant?

A. speed
B. velocity
C. momentum
D. acceleration

11. As shown in the diagram below, the river flows east at 4.2 m/s. A boat departs at 8.4 m/s relative to the water from position X.



In what direction should the boat head to reach position Y directly across the river?

A. 27° W of N
B. 30° W of N
C. 60° W of N
D. 63° W of N

12. A projectile is launched with a velocity of 23 m/s at 57° above the horizontal. What is the maximum height reached by the projectile?

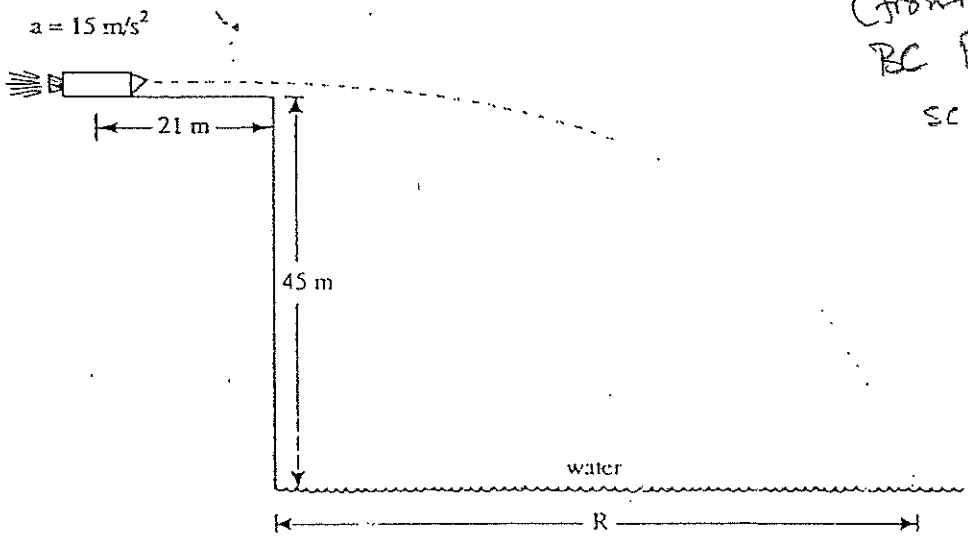
A. 8.0 m
B. 19 m
C. 27 m
D. 64 m

13. An airplane heads due west with an airspeed of 78 m/s. The wind is blowing due north at 25 m/s. What is the speed of the airplane relative to the ground?

A. 53 m/s
B. 78 m/s
C. 82 m/s
D. 103 m/s

14. A rocket accelerates at 15 m/s^2 from rest for 21 m on a frictionless horizontal surface. The rocket stops firing at the cliff and falls freely from a height of 45 m.

(from an old BC Phys 12 scholarship exam)



If air resistance is not significant, what is the distance R when the rocket hits the water?

(7 marks)

3

A ball is thrown upward from the top of a 35-m tower, Fig. 5-13, with initial velocity $v_0 = 80$ m/s at an angle $\theta = 25°$. (*a*) Find the time to reach the ground and the distance R from P to the point of impact. (*b*) Find the magnitude and direction of the velocity at the moment of impact.

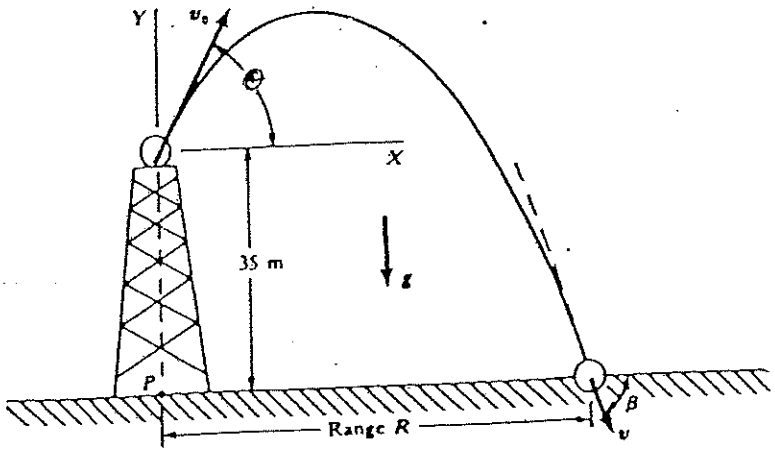
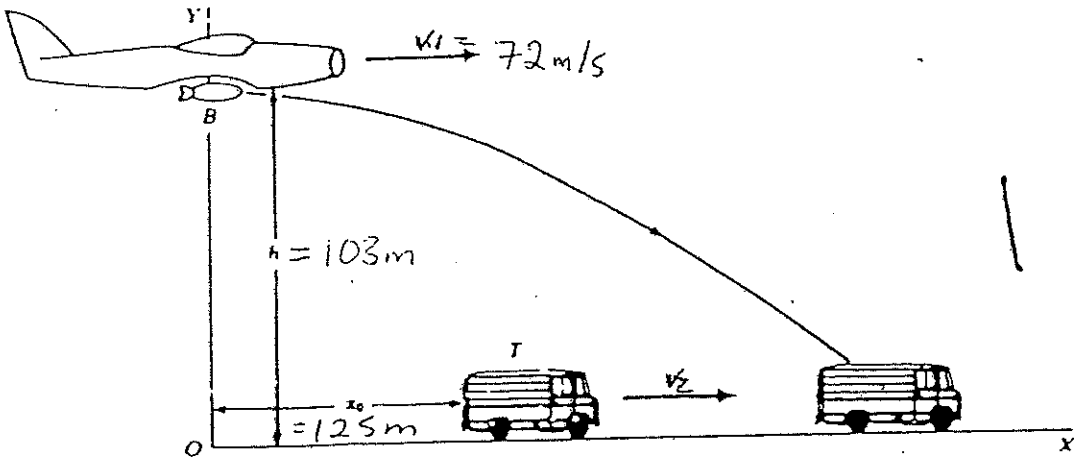


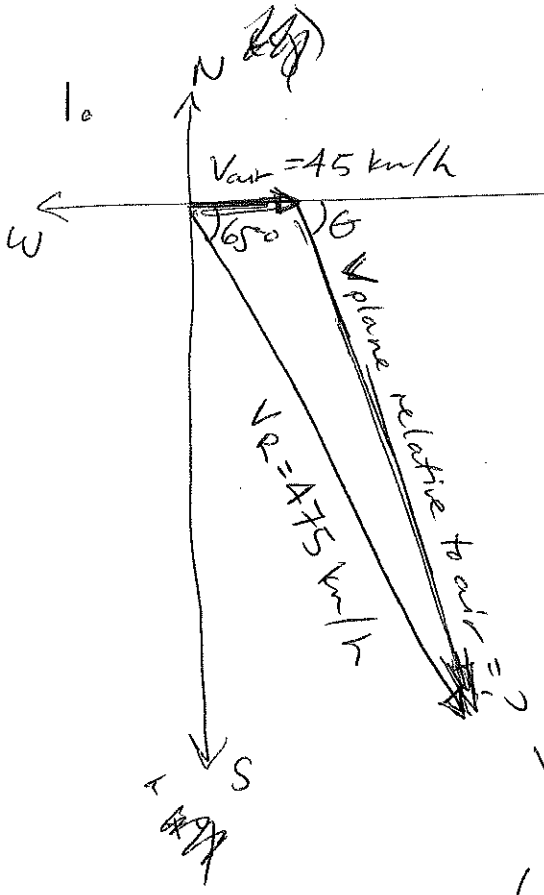
Fig. 5-13

A bomber, Fig. 5-16, is flying level at a speed $v_1 = 72$ m/s (about 161 mi/h), at an elevation of $h = 103$ m. When directly over the origin bomb B is released and strikes the truck T, which is moving along a level road (the X axis) with constant speed. At the instant the bomb is released the truck is at a distance $x_0 = 125$ m from O. Find the value of v_2 and the time of flight of B. (Assume that the truck is 3 m high.)

I The equations for x and y motion of the bomb are



Kinematics in 2-D Booklet (practice problems)

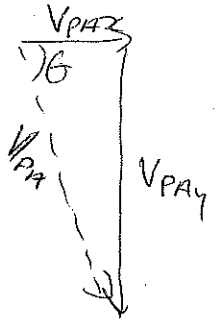


	x	y
V_{air} relative to ground	45 km/h	0
V_{plane} relative to air	$V_{PAx} = ?$	$V_{PAy} = ?$
$V_{Plane\ relative\ to\ ground}$ (resultant)	$475\cos 65^\circ$	$475\sin 65^\circ$

$$\vec{V}_{PG} = \vec{V}_{AG} + \underbrace{\vec{V}_{PA}}_{?}$$

$$x: \quad V_{PAx} = V_{PGx} - V_{AGx} = (475\cos 65^\circ - 45)$$

$$y: \quad V_{PAy} = V_{PGy} - V_{AGy} = 475\sin 65^\circ - 0$$



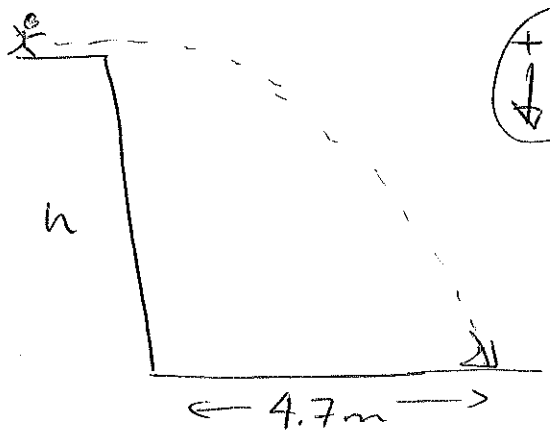
$$V_{PA} = \sqrt{(475\cos 65^\circ - 45)^2 + (475\sin 65^\circ)^2}$$

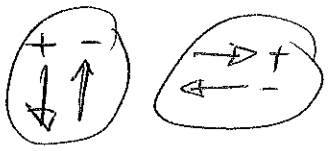
$$= 457.8 \text{ km/h}$$

$$\theta = \tan^{-1}\left[\frac{475\sin 65^\circ}{475\cos 65^\circ - 45}\right] = 70.11^\circ$$

∴ the velocity of the plane relative to the air is $\boxed{458 \text{ km/h } [70.1^\circ \text{ S of E}]}$

2.





x	y
$V_x = 2.50 m/s$	$V_{oy} = 0.0 m/s$
$d_x = 4.7 m$	$a_y = 9.8 m/s^2$
$t = ?$	$\Delta d_y = h$
	$\Delta t = ?$
	$V_{sy} = ?$

(a) Use "x" info

$$V_x = \frac{d_x}{t} \quad \therefore t = \frac{d_x}{V_x} = \frac{4.7 m}{2.5 m/s} = 1.88 s$$

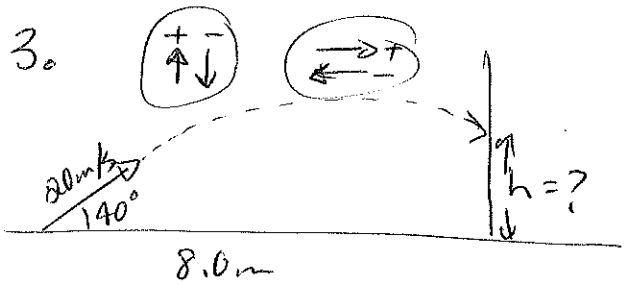
$$\boxed{t = 1.9 s}$$

(b) $\Delta d_y = \frac{1}{2} a t^2 + \cancel{V_{oy} t}^{0}$

$= \frac{1}{2}(9.8 m/s^2)(1.88 s)^2 = 17.3 m$

$\therefore \boxed{h = 17 m}$

3.



x	y
$d_x = 8.0 m$	$V_{oy} = 20 \sin 40°$
$V_x = 20.0 m/s \times \cos 40°$	$a_y = -9.8 m/s^2$
$t = ?$	$\Delta d_y = h = ?$
	$t = ?$
	$V_{sy} = ?$

x

$$t = \frac{d_x}{V_x} = \left(\frac{8}{20 \cos 40}\right)$$

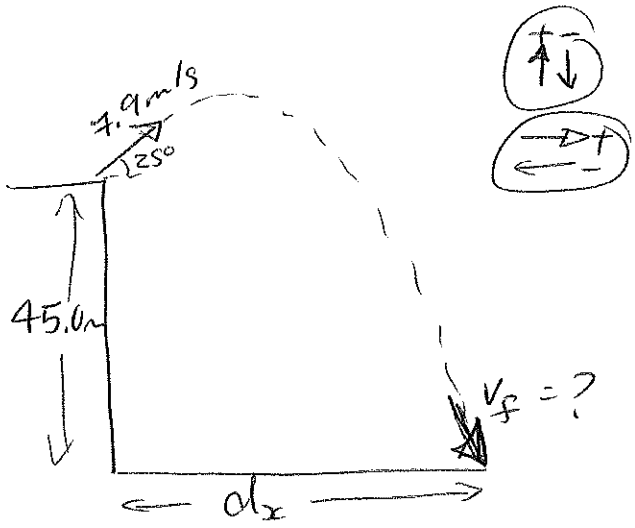
y

$$\Delta d_y = \frac{1}{2} a_y t^2 + V_{iy} t$$

$$= \frac{1}{2}(-9.8)\left(\frac{8}{20\cos 40}\right)^2 + (\cancel{20} \sin 40)\left(\frac{8}{\cancel{20}\cos 40}\right)$$

$\boxed{\Delta d = 5.4 m \text{ above the ground}}$

4.



x ///
$v_x = 7.9 \cos 25°$
$d_x = ?$
$t = ?$

y ///
$v_{oy} = (7.9 \text{ m/s}) \times (\sin 25°)$
$\Delta d = -45.0 \text{ m}$
$a_y = -9.8 \text{ m/s}^2$
$\Delta t = ?$
$v_{fy} = ?$

(a) y /// info

$\Delta d_y = \frac{1}{2} a_y t^2 + v_{oy} t$

$0 = \frac{1}{2}(-9.8) t^2 + (7.9 \sin 25°) t - (-45)$

$0 = -4.9 t^2 + (7.9 \sin 25°) t + 45$

$$t = \frac{-(7.9 \sin 25°) \pm \sqrt{(7.9 \sin 25°)^2 - 4(-4.9)(45)}}{2(-4.9)}$$

$$= \frac{-(7.9 \sin 25°) \pm 29.886}{-9.8} = 3.39 \text{ s} \quad \text{or} \quad -2.71 \text{ s}$$

$\boxed{t = 3.4 \text{ s}}$

(b) $v_{fy}^2 = 2 a_y \Delta d_y + v_{oy}^2$

$v_{fy} = \sqrt{2(-9.8)(-45) + (7.9 \sin 25°)^2}$

$= 29.89 \text{ m/s}$

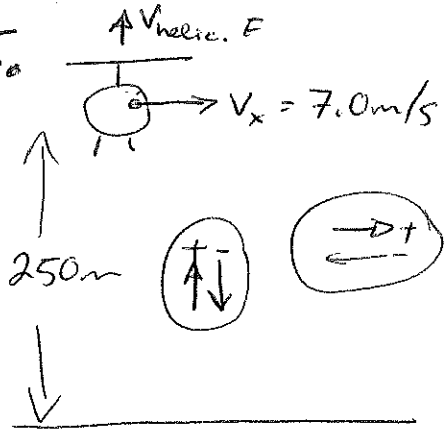
$\theta = \tan^{-1}\left[\frac{v_{fy}}{v_x}\right]$

$= 76.5°$

$v_f = \sqrt{v_x^2 + v_{fy}^2} = 30.73 \text{ m/s}$

or $\boxed{\vec{v}_f = 31 \text{ m/s} \left[77° \text{ below the horizontal}\right]}$

5.



x

$V_x = 7.0 m/s$

$d_x = ?$

$t = ?$

y

$V_{oy} = 5.0 m/s$

$a_y = -9.8 m/s^2$

$\Delta d_y = -250 m$

$\Delta t = ?$

y info $\quad \Delta d_y = \frac{1}{2} a_y t^2 + V_{oy} t$

$$0 = \tfrac{1}{2}(-9.8)t^2 + (5.0)t - (-250)$$

$$0 = -4.9t^2 + 5t + 250$$

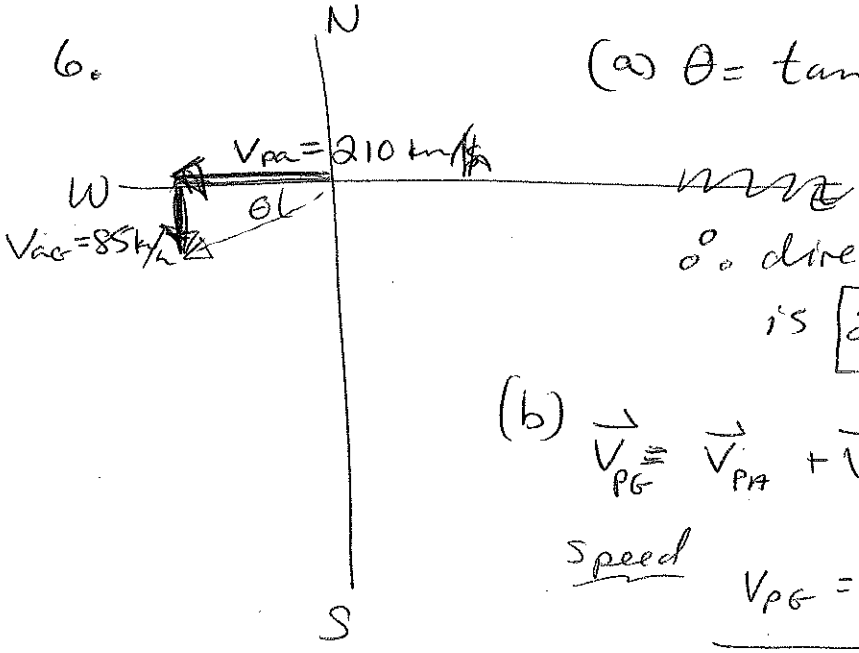
$$t = \frac{-5 \pm \sqrt{5^2 - 4(-4.9)(250)}}{2(-4.9)}$$

$$= \frac{-5 \pm 70.178}{-9.8} = 7.67 s \text{ or } -7.67 s$$

$\boxed{t = 7.7 s}$

(note the horizontal component of V_i does not impact time in the air)

6.



(a) $\theta = \tan^{-1}\left[\frac{85}{210}\right] = 22.0°$

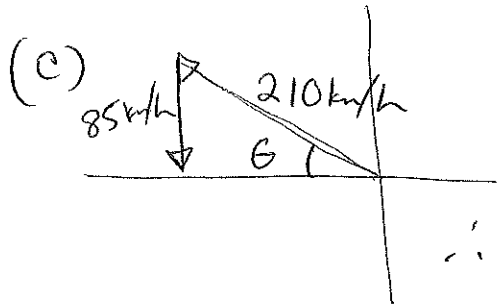
° direction of travel is **22.0° S of W**

(b) $\vec{V}_{PG} = \vec{V}_{PA} + \vec{V}_{AG}$

Speed $V_{PG} = \sqrt{85^2 + 210^2}$

$V_{PG} = 227$ km/h

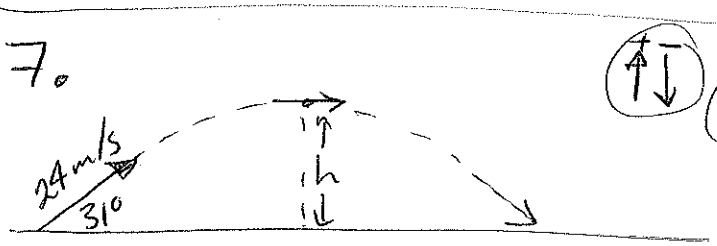
(c)

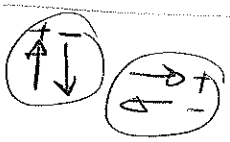


$\theta = \sin^{-1}\left[\frac{85}{210}\right] = 23.9°$

∴ heading is **23.9° N of W**

7.





x: $V_x = (24.0\text{ m/s}) \times \cos 31°$

$a_x = ?$

$t = ?$

y: $V_{oy} = (24.0\text{ m/s}) \sin 31°$

$a_y = -9.8\text{ m/s}^2$

$V_{fy} = 0.0\text{ m/s}$

$\Delta d_y = ? = h$

$t = ?$

(a) $V_{fy}^2 = 2a_y \Delta d_y + V_{oy}^2$

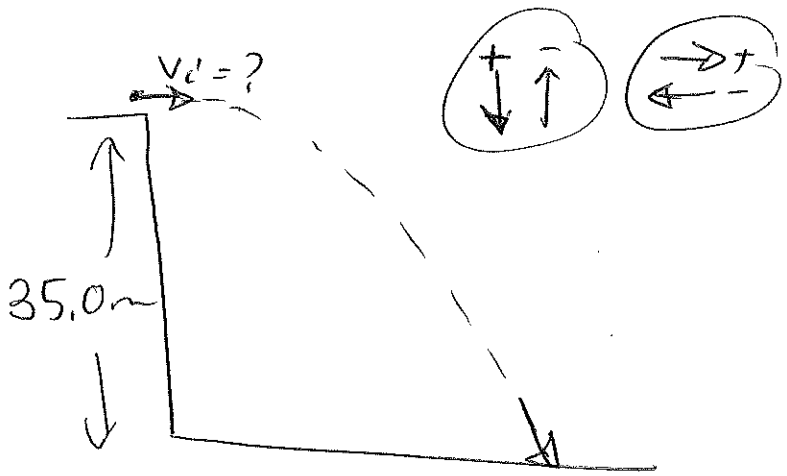
$\therefore \Delta d_y = \frac{-V_{oy}^2}{2a_y} = \frac{-(24\sin 31°)^2}{2(-9.8)} = 7.7955$

$\therefore$ **max. height = 7.80 m**

(b) at max height, $V_y = 0$ $\therefore V = V_x$

$\vec{V} = 20.6\text{ m/s}$

8.



x: $v_x = 26.0\,m/s$, $t = ?$, $d_x = ?$

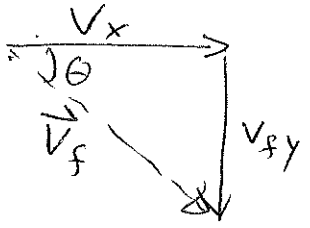
y: $v_{oy} = 0.0\,m/s$, $a_y = 9.8\,m/s^2$, $v_{fy} = ?$, $\Delta d_y = 35.0\,m$, $t = ?$

(a) y info $\Delta d_y = \frac{1}{2} a_y t^2 + \cancel{v_{oy} t}$

$$\therefore t = \sqrt{\frac{2\Delta d_y}{a_y}} = \sqrt{\frac{2(35m)}{(9.8)}}$$

$$\boxed{t = 2.67s}$$

(b) $v_{fy} = \sqrt{2 a_y \Delta d_y + \cancel{v_{oy}^2}} = \sqrt{2(9.8)(35)} = 26.19\,m/s$



$$v_f = \sqrt{v_x^2 + v_{fy}^2} = \sqrt{26^2 + (26.19)^2}$$

$$= 36.9\,m/s$$

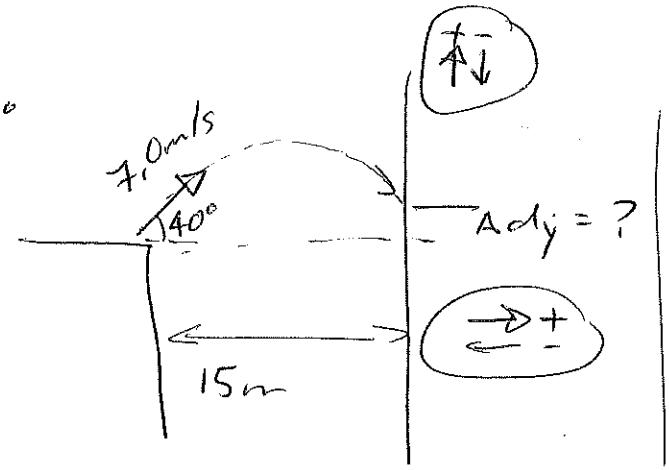
$$\theta = \tan^{-1}\left[\frac{26.19}{26}\right] = 45.2^\circ$$

$$\therefore \boxed{\vec{v}_f = 36.9\,m/s\ [45.2^\circ \text{ below the horizontal}]}$$

(c) $d_x = v_x t = (26\,m/s) \times \sqrt{\frac{2 \times 35}{9.8}}$

$$\boxed{d_x = 69.5m}$$

9.



x

$d_x = 15\,m$

$V_x = (7.0\,m/s) \times \cos 40°$

$t = ?$

y

$V_{iy} = (7.0\,m/s) \times \sin 40°$

$a_y = -9.8\,m/s^2$

$\Delta d_y = ?$

$t = ?$

$V_{fy} = ?$

x info $\quad t = \dfrac{d_x}{V_x} = \dfrac{15}{(7\cos 40°)}$

y info $\quad \Delta d_y = \frac{1}{2} a_y t^2 + V_{iy} t$

$= \frac{1}{2}(-9.8)\left(\dfrac{15}{7\cos 40°}\right)^2 + (7\sin 40°)\left(\dfrac{15}{7\cos 40°}\right)$

(-38.34)

$\Delta d_y = -25.76\,m$

∴ the ball hits the wall **26 m below** its original level.

10. D

11. B

12. B

13. C

23 m/s, 57°

$V_{oy} = 23 \sin 57°$

$V_{fy} = 0$

$a = -9.8\,m/s^2$

$\Delta d = ?$

$V_f^2 = 2a\Delta d + V_i^2$

$\Delta d = \dfrac{-V_i^2}{2a}$

14. before reaching cliff

$V_i = 0 m/s$

$\Delta d = 21 m$

$a = 15 m/s^2$

$V_f = ?$

$V_f^2 = 2a\Delta d + V_i^2$

$V_f = \sqrt{2(15)(21)}$

$= 25.0998 m/s$

---

at cliff

$+\downarrow \quad -\uparrow$

x //

$V_x = 25.0998 m/s$

$d_x = R$

$t = ?$

y //

$V_{oy} = 0.0 m/s$

$\Delta d_y = 45 m$

$a_y = 9.8 m/s^2$

$t = ?$

yinfo $\Delta d_y = \frac{1}{2} a_y t^2 + \cancel{V_{oy} t^0}$

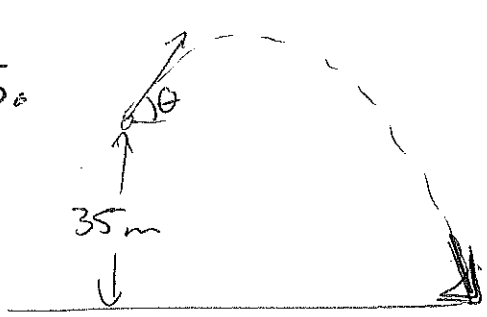
$t = \sqrt{\frac{2\Delta d_y}{a_y}}$

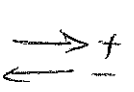
$d_x = V_x t$

$= \left[\sqrt{2(15)(21)}\right] \times \sqrt{\frac{2(45)}{9.8}}$

$d_x = \boxed{R = 76 m}$

15.





$\rightarrow$ $v_x = (80\ m/s)(\cos 25^\circ)$

$d_x = R$

$t = ?$

$\downarrow$ $v_{oy} = (80\ m/s)\sin 25^\circ$

$a_y = -9.8\ m/s^2$

$\Delta d_y = -35\ m$

$v_{fy} = ?$

$t = ?$

(a) y info

$$\Delta d_y = \tfrac{1}{2} a_y t^2 + v_{oy} t$$

$$0 = \tfrac{1}{2}(-9.8)t^2 + (80\sin 25^\circ)t - (-35)$$

$$0 = -4.9t^2 + (80\sin 25^\circ)t + 35$$

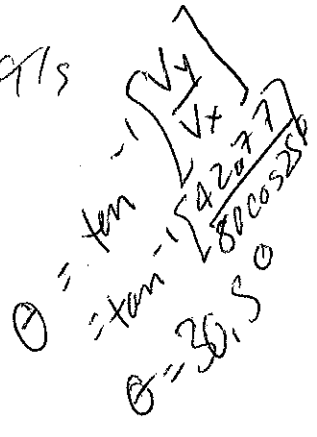
$$t = \frac{-(80\sin 25^\circ) \pm \sqrt{(80\sin 25^\circ)^2 - 4(-4.9)(35)}}{2(-4.9)}$$

$$= \frac{-33.809 \pm 42.76}{-9.8} = 7.85316 \text{ or } -0.91\ s$$

$\therefore t = 7.8\ s$

$d_x = v_x t = (80\cos 25^\circ)(7.81316)$

$d_x = R = 5.7 \times 10^2\ m$



(b) $v_{fy} = \sqrt{2 a_y \Delta d_y + v_{oy}^2} = \sqrt{2(-9.8)(-35) + (80\sin 25^\circ)^2}$

$v_{fy} = -42.77\ m/s$

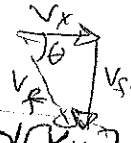
$v_f = \sqrt{v_x^2 + v_{fy}^2} = 84.2$

$\theta = \tan^{-1}\left[\frac{80\cos 25^\circ}{42.77}\right]$

$\theta = 59^\circ$

$\theta = \tan^{-1}\left[\frac{v_{fy}}{v_x}\right]$

$\theta = 30.5^\circ$



$\vec{v}_f = 84\ m/s$ [30.5° below the horizontal]

16. bomb

$\overrightarrow{x}$ $v_x = 72\,m/s$

$d_x = 125\,m + v_2 t$

$t = ?$

$\overrightarrow{y}$ $v_{oy} = 0.0\,m/s$

$a_y = 9.8\,m/s^2$

$\Delta d_y = 103\,m - 3\,m = 100\,m$

$\Delta t = ?$

$$\Delta d_y = \tfrac{1}{2} a_y t^2 + \cancel{v_{oy} t^0}$$

$$\therefore t = \sqrt{\frac{2\Delta d_y}{a_y}} = \sqrt{\frac{2(100\,m)}{9.8}} = 4.5175\,s$$

$$d_x = v_x t = (72\,m/s)(4.5175\,s)$$

$$d_x = 325.26$$

$$v_2 = \frac{d_x - 125\,m}{t} = \frac{(325.26 - 125)\,m}{(4.5175)\,s}$$

$$\boxed{v_2 = 44\,m/s}$$

$$\boxed{t = 4.5\,s}$$