CHAPTER 2

A Mathematical Toolkit

A Graphic Display

Identify the graph that shows a linear relationship.

Physics often uses mathematics as its language. This chapter presents a collection of mathematical techniques you will find useful throughout the course. You might think of the chapter as a collection of tools that can be used when needed later.

Central to the tools is the use of graphs and equations to represent the results of many observations and experiments. Probably the most famous physics equation is Einstein's $E = mc^2$. This simple equation describes one of the most powerful concepts of physics—the equivalence of mass and energy for a particle at rest.

How do you find an equation to describe experimental results? Frequently a graph of the data gives you a clue. In this chapter you will see how data describing the distance a car travels while braking to a stop can be expressed by an equation. Thus, you will learn that one of the most powerful ways of analyzing data is to display them as a graph.

Chapter Outline

2.1 THE MEASURE OF SCIENCE
- The Metric System
- Scientific Notation
- Prefixes Used With SI Units
- Arithmetic Operations in Scientific Notation

2.2 NOT ALL IS CERTAIN
- Uncertainties of Measurement
- Accuracy and Precision
- Significant Digits
- Operations Using Significant Digits

2.3 DISPLAYING DATA
- Graphing Data
- Linear, Quadratic, and Inverse Relationships

2.4 MANIPULATING EQUATIONS
- Solving Equations Using Algebra
- Units in Equations

√ Concept Check

The following terms or concepts are important for a good understanding of this chapter. If you are not familiar with them, you should review them before studying this chapter.

- algebra and graphing techniques

Objectives

- state the fundamental SI units for time, length, and mass.
- demonstrate an ability to use scientific notation.
- identify and use common metric prefixes.
- be able to perform arithmetic operations using scientific notation.

2.1 THE MEASURE OF SCIENCE

The science of physics is based on a few principles and involves the development of concepts. The application of these principles and concepts usually involves the measurement of one or more quantities. In almost every country except the United States, the metric system is used in everyday life. The world-wide scientific community uses an adaptation of the metric system, the SI, to make measurements.

The Metric System

The metric system is based on powers of ten.

The **metric system** of measurement was created by French scientists in 1795. It is convenient to use because units of different sizes are related by powers of ten. An international committee determines the standards of the metric system. This committee has set up the Système International d'Unités (**SI**). The SI is used throughout the world. Because other quantities can be described using the three basic units of the SI—time, length, and mass—they are called **base units**.

The SI unit of time, the second, is based on the oscillation time of an atom.

The standard unit of time is the **second** (s). The second was first defined as 1/86 400 of the mean solar day. A mean solar day is the average length of the day over a period of one year. In 1967, the second was redefined in terms of the frequency of one type of radiation emitted by a cesium-133 atom.

The standard SI unit of length is the **metre** (m). The metre was first defined as one ten-millionth (10^{-7}) of the distance from the north pole to the equator, measured along a line passing through Lyons, France.

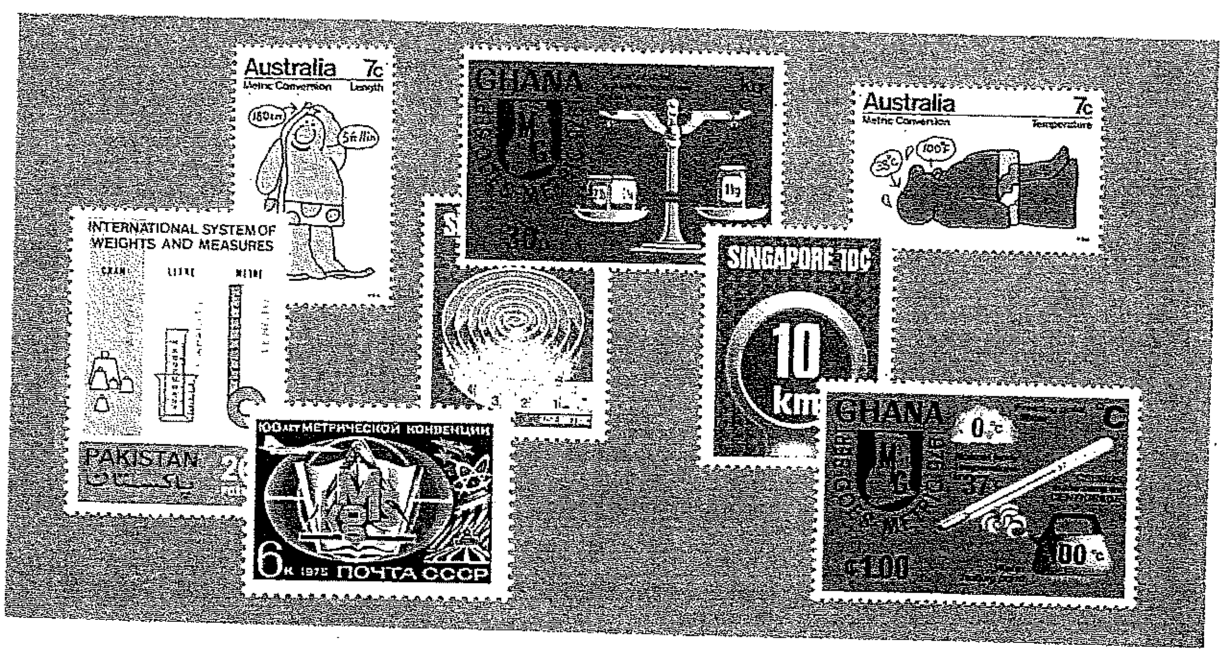


FIGURE 2–1. Several countries have issued stamps to help the general public become familiar with SI units of measurement.

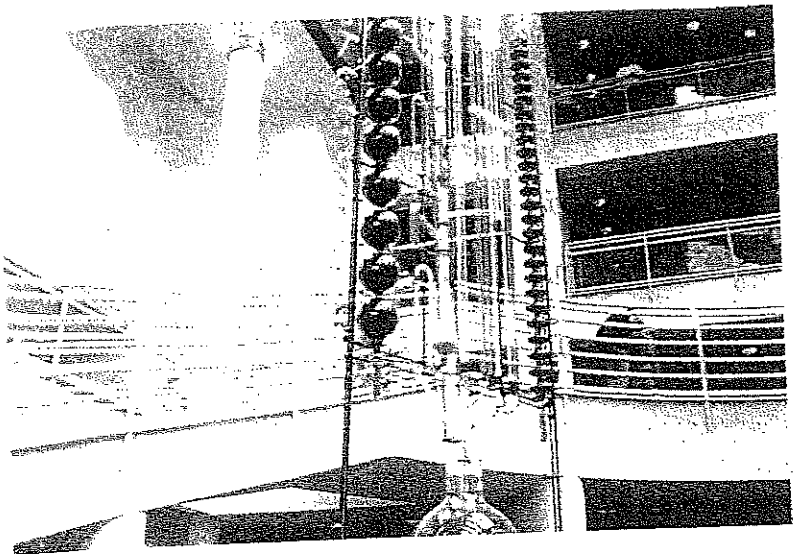


FIGURE 2–2. In ancient times, water clocks, called clepsydras, were used for keeping time.

In the 20th century, physicists found that light could be used to make very precise measurements of distances. In 1960, the metre was redefinded as a multiple of a wavelength of light emitted by krypton-86. By 1982, an even more precise length measurement defined the metre as the distance light travels in 1/229 792 458 second in a vacuum.

The SI unit of length is the metre, defined as the distance light travels in a certain amount of time.

The third standard unit measures the mass of an object. The **kilogram** (kg) is the only unit not defined in terms of the properties of atoms. It is the mass of a platinum-iridium metal cylinder kept near Paris. A copy is kept at the National Institute of Standards and Technology in the United States.

The SI unit of mass is the kilogram.

Two other base units will be introduced as needed in the text. A wide variety of other units, called **derived units**, are combinations of the base units. A common derived unit is the metre per second, or m/s, used to measure speed.

Scientific Notation

Scientists often work with very large and very small quantities. For example, the mass of Earth is about

$$6\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ \text{kg}$$

and the mass of an electron is

$$0.000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 911\ \text{kg}.$$

Written in this form, the quantities take up much space and are difficult to use in calculations. To work with such numbers more easily, we write them in a shortened form by expressing decimal places as powers of ten. This method of expressing numbers is called exponential notation. **Scientific notation** is based on exponential notation. In scientific notation, the numerical part of a measurement is expressed as a number between 1 and 10 multiplied by a whole-number power of 10.

$$M \times 10^n$$

In this expression, $1 \leq M < 10$ and n is an integer. For example, 2 km can be written 2×10^3 m. The mass of a softball is about 180 g or 1.8×10^{-1} kg.

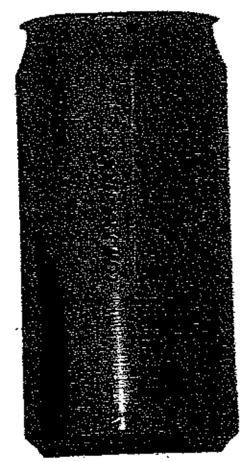


FIGURE 2–3. Most countries now use the SI unit *joule* rather than calorie for energy.

F.Y.I.

When the metric system was proposed in 1792, a 10-hour clock was included in the plan. This part of the system was never accepted and was eventually abandoned.

To write measurements using scientific notation, move the decimal point until only one non-zero digit remains on the left. Then count the number of places the decimal point was moved and use that number as the exponent of ten. Thus, the approximate mass of Earth can also be expressed as 6×10^{24} kg. Note that the exponent becomes larger as the decimal point is moved to the left.

To write the mass of the electron in scientific notation, the decimal point is moved 31 places to the right. Thus, the mass of the electron can also be written as 9.11×10^{-31} kg. Note that the exponent becomes smaller as the decimal point is moved to the right.

Practice Problems

Express the following measurements in scientific notation.

1. a. 5800 m **b.** 450 000 m **c.** 302 000 000 m **d.** 86 000 000 000 m

2. a. 0.000 508 kg **b.** 0.000 000 45 kg **c.** 0.003600 kg **d.** 0.004 kg

▶ **3. a.** 300 000 000 s **b.** 186 000 s **c.** 93 000 000 s

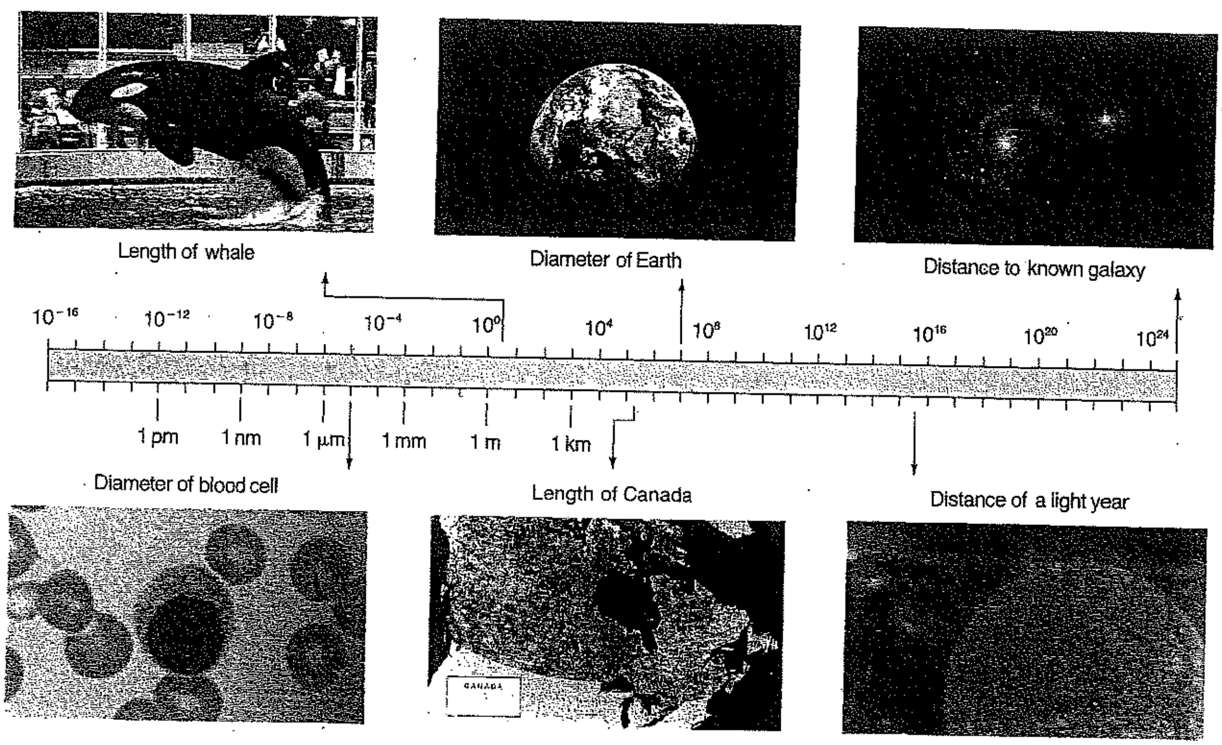
A quantity written in scientific notation consists of a number between 1 and 10 followed by 10 raised to a power.

Prefixes Used With SI Units

Like our number system, the metric system is a decimal system. **Prefixes** are used to change SI units by powers of ten. Thus, one tenth of a metre is a decimetre, one hundredth of a metre is a centimetre, and one thousandth of a metre is a millimetre. Each of these divisions can be found on a metre stick. The prefixes that change SI units by a power of one thousand are most common. Thus, one thousand metres is a kilometre. Figure 2–4 shows the vast range of lengths of objects in our universe. Commonly used length units are shown.

Metric prefixes differ from one another by a power of ten.

FIGURE 2–4. Objects in the universe range from the very small to the unimaginably large.



The metric units for all quantities use the same prefixes. One thousandth of a gram is a milligram, and one thousand grams is a kilogram. To use SI units effectively, it is important to know the meanings of the prefixes in Table 2–1.

Table 2–1

Prefixes Used with SI Units

Prefix	Symbol	Fractions	Example
pico	p	1/1 000 000 000 000 or 10^{-12}	picometre (pm)
nano	n	1/1 000 000 000 or 10^{-9}	nanometre (nm)
micro	μ	1/1 000 000 or 10^{-6}	microgram (μg)
milli	m	1/1 000 or 10^{-3}	milligram (mg)
centi	c	1/100 or 10^{-2}	centimetre (cm)
deci	d	1/10 or 10^{-1}	decimetre (dm)
		Multiples	
tera	T	1 000 000 000 000 or 10^{12}	terametre (Tm)
giga	G	1 000 000 000 or 10^{9}	gigametre (Gm)
mega	M	1 000 000 or 10^{6}	megagram (Mg)
kilo	k	1000 or 10^{3}	kilometre (km)
hecto	h	100 or 10^{2}	hectometre (hm)
deka	da	10 or 10^{1}	decagram (dag)

Example Problem

Conversion Between Units

What is the equivalent of 500 mm in metres?

Solution: From Table 2–1, we see the conversion factor is

$$1 \text{ mm} = 1 \times 10^{-3} \text{ m}$$

Therefore,

$$(500 \text{ mm}) \frac{(1 \times 10^{-3} \text{ m})}{1 \text{ mm}} = 500 \times 10^{-3} \text{ m} = 5.00 \times 10^{-1} \text{ m}$$

Practice Problems

4. Convert each of the following length measurements to its equivalent in metres.
   a. 1.1 cm **b.** 76.2 pm **c.** 2.1 km **d.** 0.123 Mm
5. Convert each of these mass measurements to its equivalent in kilograms.
   a. 147 g **b.** 11 μg **c.** 7.23 Mg **d.** 478 mg

Arithmetic Operations in Scientific Notation

Suppose you need to add or subtract measurements expressed in scientific notation. If the numbers have the same exponent, simply add or subtract the values of M and keep the same n.

Quantities to be added or subtracted must have the same exponents.

POCKET LAB

PAPER BLOCK

Look closely at the markings on a cm scale. Would you guess that a single sheet of paper has a volume of more or less than 1.0 cm^3? Estimate the volume of a single sheet of paper to one significant digit. Record your estimate in correct scientific notation. Use the cm scale to measure the volume of a sheet of paper. (**Hint:** Measure the thickness of 10 or 20 sheets.) Record your measured value. Compare the measured value to your estimate.

Using Your CALCULATOR

Using a calculator simplifies performing arithmetic operations on numbers in scientific notation.

4.0×10^{-6} kg $-$ 3.0×10^{-7} kg

Keys	Answer
[4.0] [EXP] [6] [+/-] [−]	4.-06
[3.0] [EXP] [7] [+/-] [=]	3.7-6

3.7×10^{-6} kg

$$\frac{8 \times 10^{6}\ \text{m}}{2 \times 10^{-2}\ \text{s}}$$

Keys	Answer
[8] [EXP] [6] [÷]	8.06
[2] [EXP] [2] [+/-] [=]	4.08

4×10^{8} m/s

Example Problem

Adding and Subtracting with Like Exponents

a. $4 \times 10^{8}\ \text{m} + 3 \times 10^{8}\ \text{m} = 7 \times 10^{8}\ \text{m}$

b. $6.2 \times 10^{-3}\ \text{m} - 2.8 \times 10^{-3}\ \text{m} = 3.4 \times 10^{-3}\ \text{m}$

If the powers of ten are not the same, they must be made the same before the numbers are added or subtracted. Move the decimal points until the exponents are the same.

Example Problem

Adding and Subtracting with Unlike Exponents

a. $4.0 \times 10^{6}\ \text{m} + 3 \times 10^{5}\ \text{m}$
$= 4.0 \times 10^{6}\ \text{m} + 0.3 \times 10^{6}\ \text{m} = 4.3 \times 10^{6}\ \text{m}$

b. $4.0 \times 10^{-6}\ \text{kg} - 3 \times 10^{-7}\ \text{kg}$
$= 4.0 \times 10^{-6}\ \text{kg} - 0.3 \times 10^{-6}\ \text{kg} = 3.7 \times 10^{-6}\ \text{kg}$

Suppose you have to add a measurement made in metres to one made in kilometres. You first must convert the measurements to a common unit, then make the power of ten the same. Finally you add or subtract.

Example Problem

Adding and Subtracting with Unlike Units

a. $4.1\ \text{m} + 1.5468\ \text{km} = 4.1\ \text{m} + 1546.8\ \text{m}$
$= 1550.9\ \text{m} = 1.5509\ \text{km}$

b. $2.31 \times 10^{-2}\ \text{g} + 6.1\ \text{mg} = 23.1\ \text{mg} + 6.1\ \text{mg} = 29.2\ \text{mg}$

c. $2.03 \times 10^{2}\ \text{m} + 1.057\ \text{km} = 2.03 \times 10^{2}\ \text{m} + 10.57 \times 10^{2}\ \text{m}$
$= 12.60 \times 10^{2}\ \text{m} = 1.260\ \text{km}$

Practice Problems

Solve the following problems. Express your answers in scientific notation.

6. a. $5 \times 10^{-7}\ \text{kg} + 3 \times 10^{-7}\ \text{kg}$
b. $4 \times 10^{-3}\ \text{kg} + 3 \times 10^{-3}\ \text{kg}$
c. $1.66 \times 10^{-19}\ \text{kg} + 2.30 \times 10^{-19}\ \text{kg}$
d. $7.2 \times 10^{-12}\ \text{kg} - 2.6 \times 10^{-12}\ \text{kg}$

7. a. $6 \times 10^{-8}\ \text{m}^2 - 4 \times 10^{-8}\ \text{m}^2$
b. $3.8 \times 10^{-12}\ \text{m}^2 - 1.90 \times 10^{-11}\ \text{m}^2$
c. $5.8 \times 10^{-9}\ \text{m}^2 - 2.8 \times 10^{-9}\ \text{m}^2$
d. $2.26 \times 10^{-18}\ \text{m}^2 - 1.80 \times 10^{-18}\ \text{m}^2$

▶ **8. a.** $5.0 \times 10^{-7}\ \text{mg} + 4 \times 10^{-8}\ \text{mg}$
b. $6.0 \times 10^{-3}\ \text{mg} + 2 \times 10^{-4}\ \text{mg}$
c. $3.0 \times 10^{-2}\ \text{pg} - 2 \times 10^{-6}\ \text{ng}$
d. $8.2\ \text{km} - 3 \times 10^{2}\ \text{m}$

Quantities expressed in scientific notation do not need to have the same exponents before they are multiplied or divided. Multiply the values of M, then add the exponents. The units are multiplied.

Example Problem

Multiplication Using Scientific Notation

a. $(3 \times 10^6\ \text{m})(2 \times 10^3\ \text{m}) = 6 \times 10^{6+3}\ \text{m}^2 = 6 \times 10^9\ \text{m}^2$

b. $(2 \times 10^{-5}\ \text{m})(4 \times 10^9\ \text{m}) = 8 \times 10^{9-5}\ \text{m}^2 = 8 \times 10^4\ \text{m}^2$

c. $(4 \times 10^3\ \text{kg})(5 \times 10^{11}\ \text{m}) = 20 \times 10^{3+11}\ \text{kg·m}$
$= 2 \times 10^{15}\ \text{kg·m}$

Quantities expressed in scientific notation with different exponents also can be divided. Divide the values of M, then subtract the exponent of the divisor from the exponent of the dividend.

Example Problem

Division Using Scientific Notation

a. $\dfrac{8 \times 10^6\ \text{m}}{2 \times 10^3\ \text{s}} = 4 \times 10^{(6-3)}\ \text{m/s} = 4 \times 10^3\ \text{m/s}$

b. $\dfrac{8 \times 10^6\ \text{kg}}{2 \times 10^{-2}\ \text{m}^3} = 4 \times 10^{6-(-2)}\ \text{kg/m}^3 = 4 \times 10^8\ \text{kg/m}^3$

Practice Problems

Find the value of each of the following quantities.

9. a. $(2 \times 10^4\ \text{m})(4 \times 10^8\ \text{m})$ **c.** $(6 \times 10^{-4}\ \text{m})(5 \times 10^{-8}\ \text{m})$

b. $(3 \times 10^4\ \text{m})(2 \times 10^6\ \text{m})$ **d.** $(2.5 \times 10^{-7}\ \text{m})(2.5 \times 10^{16}\ \text{m})$

10. a. $\dfrac{6 \times 10^8\ \text{kg}}{2 \times 10^4\ \text{m}^3}$ **c.** $\dfrac{6 \times 10^{-8}\ \text{m}}{2 \times 10^4\ \text{s}}$

b. $\dfrac{6 \times 10^8\ \text{kg}}{2 \times 10^{-4}\ \text{m}^3}$ **d.** $\dfrac{6 \times 10^{-8}\ \text{m}}{2 \times 10^{-4}\ \text{s}}$

▶ **11. a.** $\dfrac{(3 \times 10^4\ \text{kg})(4 \times 10^4\ \text{m})}{6 \times 10^4\ \text{s}}$ **b.** $\dfrac{(2.5 \times 10^6\ \text{kg})(6 \times 10^4\ \text{m})}{5 \times 10^{-2}\ \text{s}^2}$

CONCEPT REVIEW

1.1 Some calculators display large numbers as 1.574 E8. Express in normal scientific notation.

1.2 Your height might be given either in terms of a small unit, like a millimetre, or a larger unit, like a metre. In which case would your height be a larger number?

1.3 Describe in detail how you would measure the time in seconds needed to go from home to school.

1.4 **Critical Thinking:** What additional steps would you need to time your trip, using one clock at home and one at school?

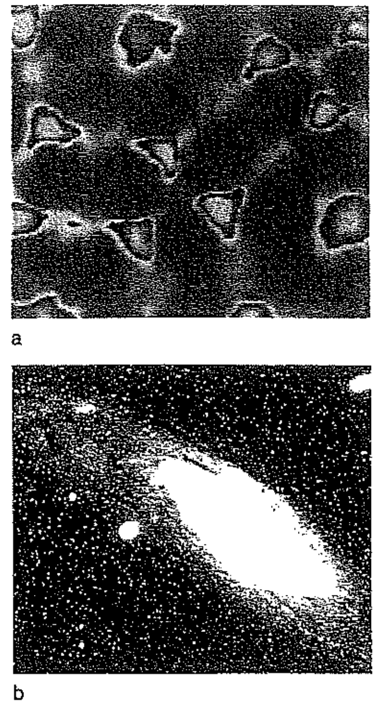


FIGURE 2–5. For extremely small measurements, such as the diameter of a red blood cell (a), and for very large measurements, such as the distance to the Andromeda galaxy (b), it is convenient to use scientific notation. Showing all the digits in numbers like these makes calculations difficult.

The product of two numbers written in scientific notation is the product of the values of M times 10 raised to the sum of their exponents.

The quotient of two numbers is the quotient of the values of M times 10 raised to the difference of their exponents.

2.2 NOT ALL IS CERTAIN

Objectives

- recognize that all measured quantities have uncertainties.
- distinguish between accuracy and precision.
- show you can use significant digits and that you understand their use in stating the precision of measured quantities.
- use significant digits correctly when recording measured data.

Often several scientists measure the same quantities and compare the data they obtain. Each scientist must know how trustworthy the data are. Every measurement, whether made by a student or a professional scientist, is subject to uncertainty.

The length of a ruler can change with changes of temperature. An electric measuring device can be affected by magnetic fields near it. In one way or another, all instruments are subject to external influences. Uncertainties in measurement cannot be avoided, although we try to make them as small as possible. For this reason, it is important to clearly describe the uncertainties in our measurements.

Uncertainties of Measurements

All measurements are subject to uncertainties.

In addition to uncertainties due to external causes, such as those listed above, the accuracy of a measurement is affected by the person making the reading. One common source of error comes from the angle at which an instrument is read. In a car, the passenger's reading of the gas gauge and the driver's reading of the same gauge can be quite different. From the passenger's viewpoint, Figure 2–6, the needle is on the second division. From the driver's seat, the needle is above the third division. The driver's reading is more correct. The difference in the readings is caused by parallax. **Parallax** (PAR uh laks) is the apparent shift in the position of an object when it is viewed from various angles. The relative positions of the object and a reference point behind it change. Gas gauges and laboratory instruments must be read at eye level and straight on to avoid parallax errors.

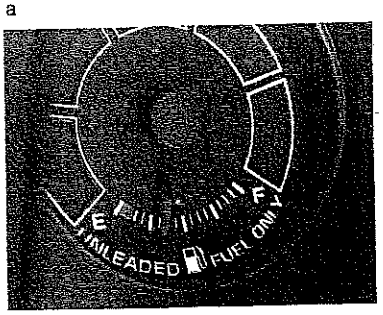
Precision is the degree of exactness to which a measurement can be reproduced.

Accuracy and Precision

Precision is the degree of exactness to which the measurement of a quantity can be reproduced. For example, a student was conducting an experiment to determine the speed of light. Several trials were made that yielded values ranging from 3.000×10^8 m/s to 3.002×10^8 m/s, with an average of 3.001×10^8 m/s. This led the student to report that the speed of light is $(3.001 \pm 0.001) \times 10^8$ m/s. According to the student's measurements, the speed of light might range from 3.000×10^8 m/s to 3.002×10^8 m/s. The precision of the measurement was 0.001×10^8 m/s.

The precision of an instrument is limited by the smallest division on the measurement scale.

a



b

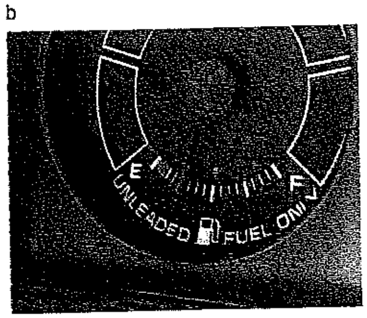


FIGURE 2–6. A parallax example is shown when a car's gasoline gauge is viewed from the passenger's seat (a) and the driver's seat (b). Note the apparent difference in readings.

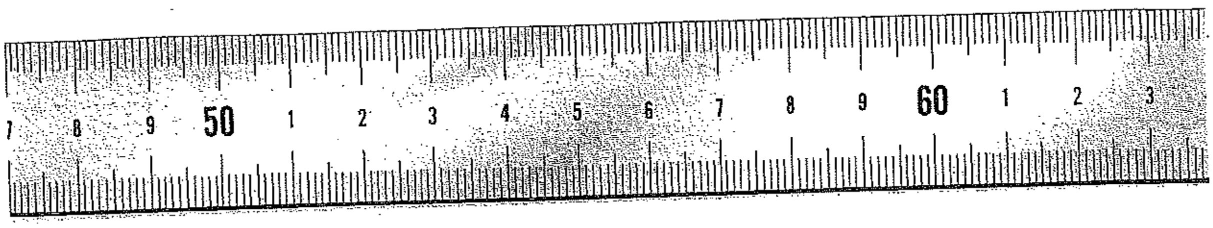


FIGURE 2–7. The metre stick contains decimetre, centimetre, and millimetre divisions.

The precision of a measuring device is limited by the finest division on its scale. The smallest division on a metre stick, Figure 2–7, is a millimetre. Thus, a measurement of any smaller length with a metre stick can be only an estimate. Even on a micrometer, Figure 2–8a, you can make measurements to only 5 μm. There is a limit to the precision of even the best instruments.

The accuracy of a measurement describes how well the result agrees with an accepted value.

Accuracy is the extent to which a measured value agrees with the standard value of a quantity. In the experiment to measure the speed of light, the accuracy is the difference between the student's measurement and the defined value of the speed of light, quoted to the same precision: 2.998×10^8 m/s. Thus, the accuracy is $(3.001 \times 10^8 \text{ m/s}) - (2.998 \times 10^8 \text{ m/s}) = 0.003 \times 10^8$ m/s. It would be possible to make a very precise measurement because the instrument is very sensitive, but have that measurement be inaccurate because the instrument was uncalibrated or because you made a reading error.

Significant digits are all the digits that are certain plus a digit that estimates the fraction of the smallest division of the measuring scale

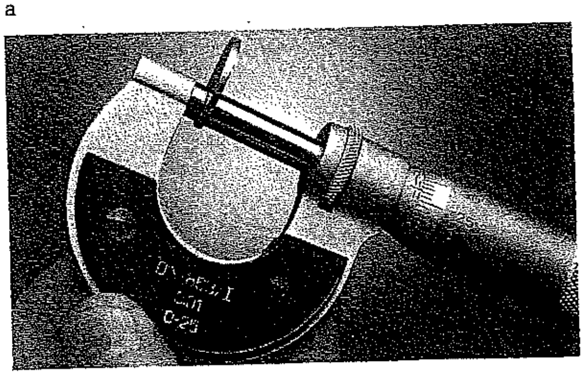
The accuracy of an instrument depends on how well its performance compares to a currently accepted standard. The accuracy of measuring devices should be checked regularly. They can be calibrated by using them to measure quantities whose values are accurately known. Uncertainties in measurement affect the accuracy of a measurement. Precision, however, is not affected because it is based on the smallest division on the instrument.

Significant Digits

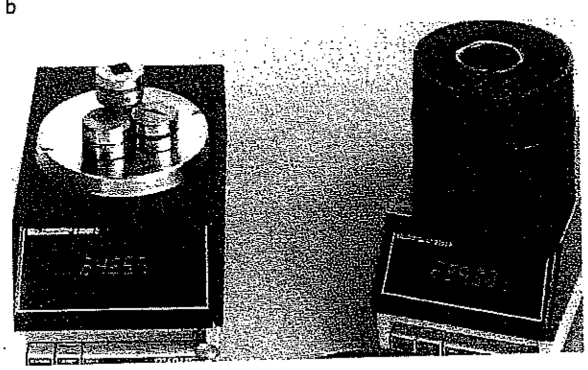
Because the precision of all measuring devices is limited, the number of digits that are valid for any measurement is also limited. The valid digits are called the **significant digits**. Suppose you measure the length

FIGURE 2–8. The micrometer (a) and analytical balance (b) are used to obtain very precise measurements of length and mass, respectively.

a



b



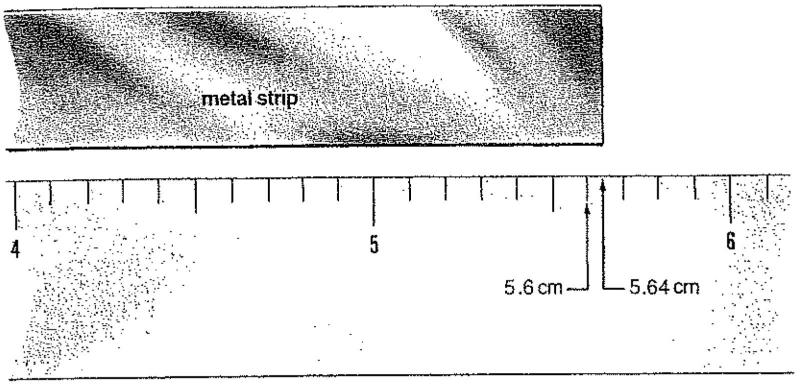


FIGURE 2–9. The accuracy of any measurement depends on both the instrument used and the observer. After a calculation, keep only those digits that truly reflect the accuracy of the original measurement.

of a strip of metal with a metre stick. The smallest division on the metre stick is a millimetre. You should read the scale to the nearest millimetre and then estimate any remaining length as a fraction of a millimetre. The metal strip in Figure 2–9 is somewhat longer than 5.6 cm (56 mm). Looking closely at the scale, you can see that the end of the metal strip is about four-tenths of the way between 56 and 57 mm. Therefore, the length of the strip is best stated as 56.4 mm. The last digit is an estimate. It might not be 4 but is likely not larger than 5 or smaller than 3. Your measurement, 56.4 mm, contains three significant digits. They are the two digits you are sure of, 5 and 6, and one, 4, that is an estimated digit.

Suppose that the end of the metal strip is exactly on the 56 mm mark. In this case, you should record the measurement as 56.0 mm. The zero indicates that the strip is not 0.1 mm more or less than 56 mm. The zero is a significant digit because it transmits information. It is the uncertain digit because you are estimating it. The last digit given for any measurement is the uncertain digit. *All non-zero digits in a measurement are significant.*

Write results in scientific notation to indicate clearly which zeros are significant.

Zeros are often a problem. The zero mentioned in 56.0 mm is significant. However, a zero that only serves to locate the decimal point is not significant. Thus the value of 0.0026 kg contains two significant digits. The measurement of 0.002060 kg contains four significant digits. The final zero indicates a reasonable estimate.

There is no way to tell how many of the zeros in the measurement 186 000 m are significant. The 6 may have been the estimated digit and the three zeroes may be needed only to place the decimal point. Or, all three zeros may be significant because they were measured. To avoid confusion, such measurements are written in scientific notation. In the number that appears before the power of ten, all the digits are significant. Thus, 1.860×10^5 m has four significant digits. To summarize, these rules are used to determine the number of significant digits.

1. Nonzero digits are always significant.
2. All final zeros after the decimal point are significant.
3. Zeros between two other significant digits are always significant.
4. Zeros to the right of a whole number are considered to be ambiguous.

F.Y.I.

Any error that can creep in, will. It will be in the direction that will do most damage to the calculation.

Murphy's law

Practice Problems

12. State the number of significant digits in each measurement.
    a. 2804 m
    b. 2.84 m
    c. 0.0029 m
    d. 0.003 068 m
    e. 4.6×10^5 m
    f. 4.06×10^5 m

13. State the number of significant digits for each measurement.
    a. 75 m
    b. 75.00 mm
    c. 0.007 060 kg
    d. 1.87×10^6 ml
    e. 1.008×10^8 m
    f. 1.20×10^{-4} m

Operations Using Significant Digits

The sum or difference of two values is as precise as the least precise value.

When you are working in the laboratory it is extremely important to remember that *the result of any mathematical operation with measurements can never be more precise than the least precise measurement.* Suppose you measure the lengths 6.48 m and 18.2 m. You are asked to find the sum of the two lengths. The length 18.2 m is precise only to one-tenth of a metre. The result of any mathematical operation with measurements cannot be more precise than the least precise measurement. Therefore, the sum of the two lengths can be precise only to one-tenth of a metre. First add 6.48 m to 18.2 m to get 24.68 m. Then round off the sum to the nearest tenth of one metre. The correct value is 24.7 m. Subtraction is handled the same way. To add or subtract measurements, first perform the operation, and then round off the result to correspond to the least precise value involved.

Example Problem

Significant Digits—Addition and Subtraction

Add 24.686 m + 2.343 m + 3.21 m.

Solution:

$$\begin{array}{r} 24.686\ \text{m} \\ 2.343\ \text{m} \\ \underline{3.21\ \ \text{m}} \\ 30.239\ \text{m} \\ = 30.24\ \ \text{m} \end{array}$$

Note that 3.21 m is the least precise measurement. Round off the result to the nearest hundredth of one metre. Follow the same rules for subtraction.

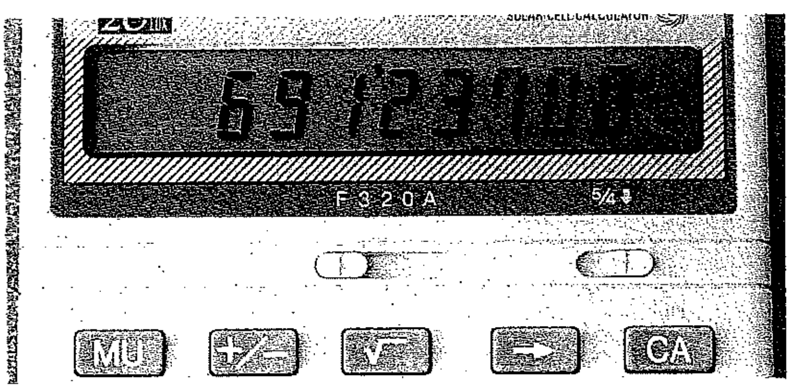


FIGURE 2–10. When using a calculator in solving problems, it is important to note that your answers cannot be more precise than the least precise quantity involved.

A different method is used to find the correct number of significant digits when multiplying or dividing measurements. After performing the calculation, note the factor with the least number of significant digits. Round the product or quotient to this number of digits.

Example Problem

Significant Digits—Multiplication

Multiply 3.22 cm by 2.1 cm.

Solution:

$$\begin{array}{r l} 3.2\,\textcircled{2} & \text{cm} \\ \underline{2.\,\textcircled{1}} & \underline{\text{cm}} \\ \textcircled{3}\,\textcircled{2}\,\textcircled{2} & \\ \underline{6\;4\;\textcircled{4}\;\;} & \\ 6.\textcircled{7}\,\textcircled{6}\,\textcircled{2} & \text{cm}^2 \end{array}$$

This is correctly stated as 6.8 cm^2.

The number of significant digits in a product or quotient is the number in the factor with the lesser number of significant digits.

The less precise factor, 2.1 cm, contains two significant digits. Therefore the product has only two. Note that each circled digit is doubtful, either because it is an estimated measurement or is multiplied by an estimated measurement. Since the 7 in the product is doubtful, the 6 and 2 are certainly not significant. The answer is best stated as 6.8 cm^2.

Example Problem

Significant Digits—Division

Divide 36.5 m by 3.414 s.

Solution:

$$\frac{36.5\ \text{m}}{3.414\ \text{s}} = 10.69\ \text{m/s}$$

This is correctly stated as 10.7 m/s.

Be sure to record all measurements made during an experiment with the correct number of significant digits. The number of significant digits shows the precision of the instrument. When using a calculator, be particularly careful to record your answer with the proper number of significant digits, even though the calculator shows additional, meaningless digits, Figure 2–10.

It is important to understand that significant digits are only considered when calculating with measurements, not when counting. For example, if you add 2 pencils and 3 pencils, you will have exactly 5 pencils, with no uncertainty. Because of the uncertainty in all measurements, it is important to let anyone who is using your data know exactly how precise your values are.

POCKET LAB

HIGH AND LOW

Measure the height of the tallest person and the shortest person in your lab group to the nearest 0.1 cm. Estimate the uncertainty in each measurement. Predict how tall each person would be from heel to the top of the head when lying down on the lab table. Would the values be within the uncertainty of the first measurement? Try it. Explain the results.

Practice Problems

14. Add 6.201 cm, 7.4 cm, 0.68 cm, and 12.0 cm.

15. Subtract

- **a.** 8.264 g from 10.8 g.
- **b.** 0.4168 m from 475 m.

16. Perform the following multiplications.
   a. 131 cm × 2.3 cm
   b. 3.2145 km × 4.23 km

▶ 17. Perform the following divisions.
   a. 20.2 cm ÷ 7.41 s
   b. 3.1416 cm ÷ 12.4 s

CONCEPT REVIEW

2.1 If you have a micrometer that has been bent more than 1 mm out of alignment, how would it compare to a new, quality metre stick in precision? accuracy?

2.2 Does parallax affect the precision of a measuring instrument? Explain.

2.3 Explain in your own words the range in heights that would be implied if you reported a measurement as "His height is 182 cm."

2.4 **Critical Thinking:** Poll takers often interview people and report "1000 people were interviewed for a margin of accuracy of ±3%." Is this a measure of the precision or accuracy of the poll? **Hint:** Suppose you were interviewed by your principal about your opinions about school rules.

2.3 DISPLAYING DATA

Objectives

- **distinguish between dependent and independent variables.**
- **be able to graph data points.**
- **understand how smooth curves drawn through data points represent the relationship between independent and dependent variables.**
- **recognize linear and direct relationships and be able to find and interpret the slope of the curve.**
- **recognize quadratic and inverse relationships.**

A well-designed graph is more than a "picture worth a thousand words." It can give you more information than either words, columns of numbers, or equations. To be useful, however, a graph must be drawn properly. In this section we will develop the use of graphs to display data.

Graphing Data

One of the most important skills to learn in driving a car is how to stop it safely. No car can "stop on a dime." The faster the car is going, the farther it travels before it stops. If you studied for a driver's licence, you probably found a table in the manual showing how far a car moves beyond the point at which the driver makes a decision to stop.

One manual shows that a car travels a certain distance between the time the driver decides to stop the car and the time the brakes are applied. This is called the "reaction distance." When the brakes are applied, the car slows down and travels the "braking distance." Table 2–2 and the bar graph, Figure 2–11, show these distances, together with the total stopping distance for various speeds.

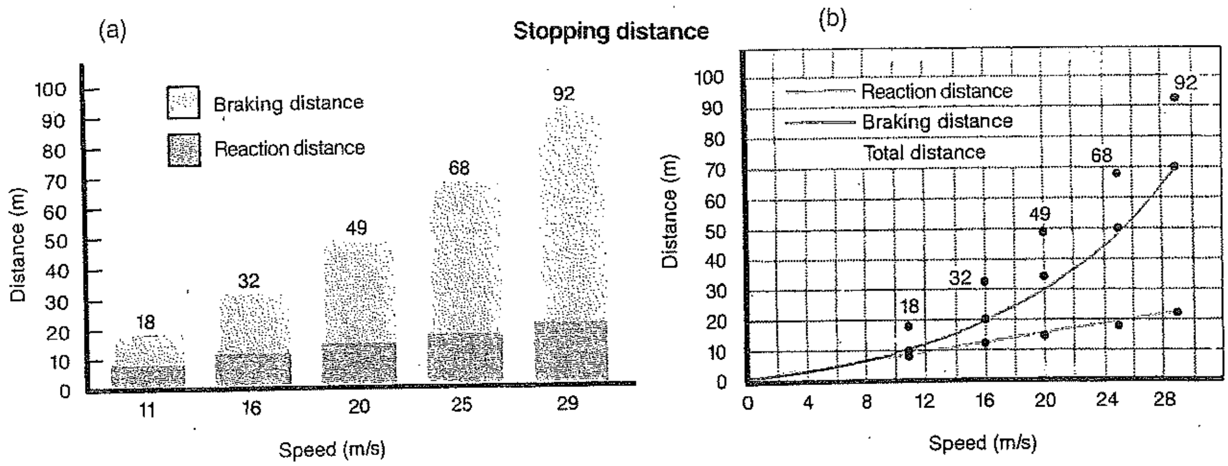


FIGURE 2–11. The total stopping distance is the sum of the reaction and braking distances. Graphs (a) and (b) display the same information in two different ways.

Table 2–2

Reaction and Braking Distances vs Speed

Original speed m/s	Reaction distance m	Braking distance m	Total distance m
11	8	10	18
16	12	20	32
20	15	34	49
25	18	50	68
29	22	70	92

The first step in analyzing data is to look at them carefully. Which variable did the experimenter (the driver) change? In our example, it was the speed of the car. Thus speed is the **independent variable**, or manipulated variable. The other two variables, reaction distance and braking distance, changed as a result of the change in the speed. These quantities are called **dependent variables**, or responding variables.

The independent variable is the one the experimenter can control directly. The value of the dependent variable depends on the independent variable.

How do the distances change for a given change in the speed? Notice that the reaction distance increases by about the same amount for each increase in speed. The braking distance, however, increases much more as the speed increases. The relationship between the distances and speed can be seen more easily if the data are plotted on a graph. To avoid confusion, you should plot two graphs, one of reaction distance and the other of braking distance.

PROBLEM SOLVING STRATEGY

Plotting Graphs

These steps will help you plot graphs from data tables.

1. Identify the independent and dependent variables. The independent variable is plotted on the horizontal, or x-axis. The dependent variable is plotted on the vertical, or y-axis. In our example, speed is plotted on the x-axis, and distance is plotted on the y-axis.

FIGURE 2–12. Knowing when and how to apply brakes safely is an important part of a driver's test.

2. Determine the range of the independent variable to be plotted. In the example, data are given for speeds between 11 and 29 m/s. A convenient range for the x-axis might be 0–30 m/s.
3. Decide if the origin (0, 0) is a valid data point. When the speed is zero, reaction and stopping distances are obviously both zero. In this case then, your graph should include the origin. Spread out the data as much as possible. Let each space on the graph paper stand for a convenient unit. Choose 2, 5, or 10 spaces to represent 10 m/s.
4. Number and label the horizontal axis.
5. Repeat steps 2–4 for the dependent variable.
6. Plot your data points on the graph.
7. Draw the best straight line or smooth curve that passes through as many data points as possible. *Do not use a series of straight line segments that "connect the dots."*
8. Give the graph a title that clearly tells what the graph represents.

Linear, Quadratic, and Inverse Relationships

The graph, Figure 2–13, of reaction distance versus speed is a straight line. That is, the dependent variable varies linearly with the independent variable; there is a **linear relationship** between the two variables.

The relationship of the two variables shown in this graph can be written as an equation

$$y = mx + b,$$

where m and b are constants called the slope and y-intercept, respectively. Each constant can be found from the graph. The **slope**, m, is the ratio of the vertical change to the horizontal change. To find the slope, select two points, **A** and **B**, as far apart as possible on the line. *They should not be data points.* The vertical change, or rise, Δy, is the difference in the vertical values of **A** and **B**. The horizontal change, or run, Δx, is the difference in the horizontal values of **A** and **B**. The slope of the graph is then calculated as

$$m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{(20 - 0)\text{ m}}{(27 - 0)\text{ m/s}} = \frac{20\text{ m}}{27\text{ m/s}} = 0.74\text{ s}.$$

Notice that units have been kept with the variables.

A Graphic Display

The independent variable is plotted on the x-axis. The dependent variable is plotted on the y-axis.

The slope of a graph is $\Delta y/\Delta x$.

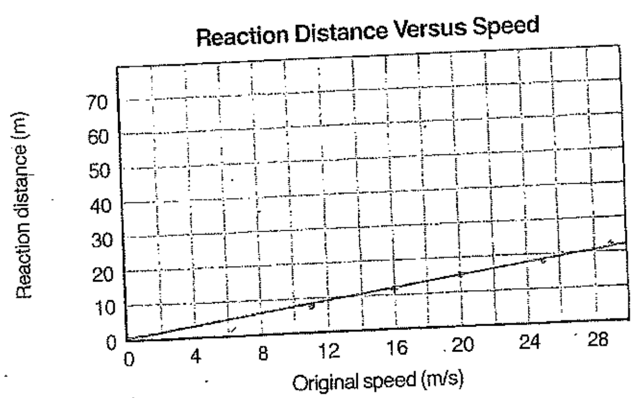
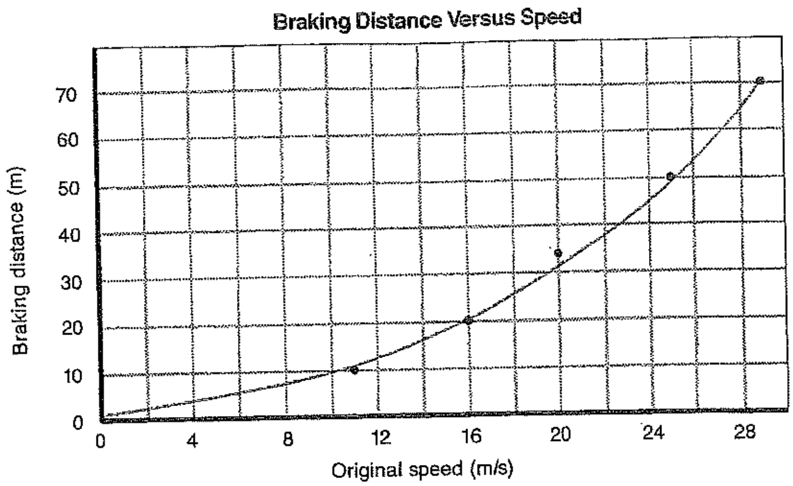


FIGURE 2–13. The graph indicates a linear relationship between reaction distance and car speed.

FIGURE 2–14. The graph indicates a parabolic relationship; braking distance varies as the square of the original speed.



The y-intercept is the value of y when x is zero.

If the dependent variable decreases when the independent variable increases, then the slope is negative.

POCKET LAB

JUICED UP

Use a sharp pencil to make marks at 2.0 cm, 4.0 cm, and 6.0 cm on the insides of a juice can. Fill the can to the 2.0 cm mark and measure the mass of the can and water on a balance. Fill the can to the 6.0 cm mark and again measure the mass on a balance. Make a graph of mass (vertical) versus height (horizontal). Predict the mass when the can has 4.0 cm of water. (**Hint:** The graph should be a straight line.) Explain why the graph does not go through 0,0. What is the meaning of the slope of the graph?

The ***y*-intercept,** *b*, is the point at which the line crosses the *y*-axis, and is the *y* value when the value of *x* is zero. In this case, when *x* is zero, the value of *y* is 0 m. For special cases like this, when the *y*-intercept equals zero, the equation becomes $y = mx$. The quantity *y* varies directly with *x*.

The value of *y* does not always increase with increasing *x*. If *y* gets smaller as *x* gets larger, then $\Delta y/\Delta x$ is less than zero, and the slope is negative.

After drawing the graph and obtaining the equation, check to see if the results make sense. The slope indicates the increased reaction distance for an increase in speed. It has units of meters/(meters/second), or seconds. Thus it is a time, the reaction time. It is the time your body takes from the instant the message to stop the car registers in your brain until your foot hits the brake pedal. Thus, the slope is a *property* of the system the graph describes.

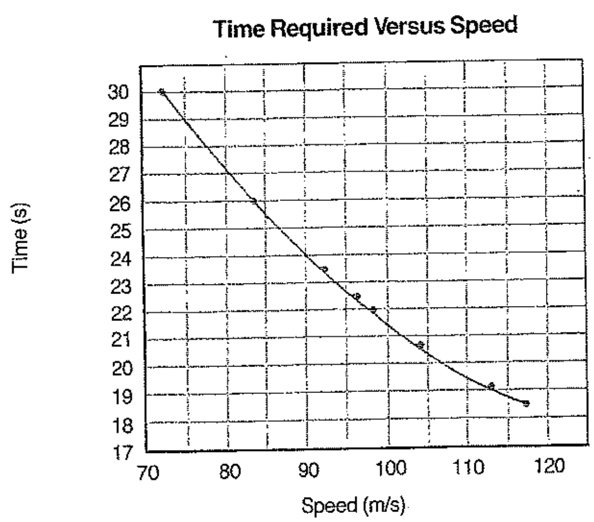


FIGURE 2–15. The graph shows the inverse relationship between the time required to travel a fixed distance and the speed.

Figure 2–14 is the graph of the braking distance versus speed, completed as suggested in the Problem Solving Strategy. Note that the relationship is not linear.

The smooth line drawn through all the data points curves upward. You cannot draw a straight line through all the points. Such graphs are frequently parabolas, indicating that the two variables are related by the equation

$$y = kx^2,$$

where k is a constant. This equation shows that y varies directly with the square of x. This equation is one form of a **quadratic relationship**. The constant k shows how fast y changes with x^2. In Chapters 4 and 5, we will discuss variables that are related by this equation, and learn why braking distance depends on speed in this way.

Some variables are related by the type of graph shown in Figure 2–15. In this case, a plot has been made of the time required to travel a fixed distance as the speed of travel is changed. When the speed is doubled, the time is reduced to one-half its original time. The relationship between speed and time is an inverse variation. The graph is a hyperbola, not a straight line. The general equation for an **inverse relationship** is

$$xy = k \text{ or } y = k \cdot \left(\frac{1}{x}\right) = kx^{-1}.$$

Circumference Versus Diameter

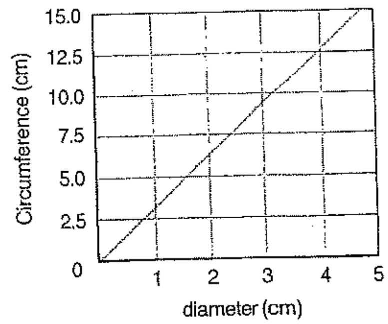


FIGURE 2–16. Use with Concept Review 3.4.

The parabolic relationship exists when one variable depends on the square of another.

CONCEPT REVIEW

3.1 What would be the meaning of a non-zero y-intercept to the graph of reaction distance versus speed?

3.2 At what speed is the reaction distance 10 m?

3.3 Explain in your own words the significance of a steeper line, or larger slope to the graph of reaction distance versus speed.

3.4 Critical Thinking: Figure 2–16 shows the relationship between the circumference and the diameter of a circle. Could a different circle be described by a different straight line? What is the meaning of the slope?

A hyperbola results when one variable depends on the inverse of the other.

2.4 MANIPULATING EQUATIONS

The manner in which one quantity, such as the circumference of a circle, depends on another, such as the circle's diameter, can be represented symbolically by an equation, $C = \pi d$, as well as by a graph. If we want to find how the diameter depends on the circumference, we can use rules of algebra to rearrange the equation, $d = C/\pi$.

Objectives

- **demonstrate the ability to manipulate algebraic equations.**
- **use dimensional analysis to test the validity of an equation.**

HELP WANTED

ACTUARY

A logical, analytical mind and exceptional knowledge of math and statistics are required for the position of actuary in the corporate headquarters of this major insurance company. A university or college background as well as solid oral and written communication skills are required; and certification on appropriate examinations is a must. For information contact:

Canadian Institute of Actuaries
360 Albert St., Suite 1040
Ottawa, Ontario, K1R 7X7

Solve equations for the required variable, the unknown, placing it on the left-hand side of the equals sign.

independent - horizon
dependent - vertical

Solving Equations Using Algebra

From your graph of the reaction distance versus speed, you can obtain an equation relating the two variables. If we represent the reaction distance by d, the speed of the car by v, and the slope, which we discovered had units of time, by t, the equation is

$$d = vt, \text{ or } d = tv.$$

Note that this last equation has the same form, but different symbols than $y = mx$. The distance, d, is the dependent variable. The speed, v, is the independent variable. The slope, m, is the reaction time, t. You can use this equation to find d if you know v and t. What if, however, you know d and t and want to find v? You can solve the equation above for v. First, place the term containing v on the left side.

$$vt = d$$

To get v alone on the left side, but not change the value of the relationship, divide both sides of the equation by t. Thus,

$$v = \frac{d}{t}.$$

That is, the speed of the car is equal to the reaction distance divided by the reaction time. If an equation contains several factors, the same process is followed until the unknown is isolated on the left side of the equation. The steps can be performed in any sequence; just be sure you perform the same operations on both sides of the equation.

Example Problem

Solving Equations

Solve the following equation for x.

$$\frac{ay}{x} = \frac{cb}{s}$$

Solution:
Multiply both sides by x.

$$ay = \frac{cbx}{s}$$

Rearrange to bring x to the left side.

$$\frac{cbx}{s} = ay$$

Divide both sides by cb.

$$\frac{x}{s} = \frac{ay}{cb}$$

Multiply both sides by s.

$$x = \frac{asy}{cb}$$

F.Y.I.

"Mathematics is the door and the key to the sciences."
Roger Bacon

Example Problem

Solving Equations

Solve the following equation for x. $y = mx + b$

Solution:

Rearrange to bring x to the left side. $mx + b = y$

Subtract b from both sides. $mx = y - b$

Divide both sides by m. $x = (y - b)/m$

Practice Problems

18. Solve the following equation for b. $y = mx + b$

19. Solve the following equations for v.

a. $d = vt$ **c.** $a = \frac{v^2}{2d}$

b. $t = \frac{d}{v}$ **d.** $\frac{v}{a} = \frac{b}{c}$

20. Solve each of these equations for E.

a. $f = \frac{E}{s}$ **b.** $m = \frac{2E}{v^2}$ **c.** $\frac{E}{c^2} = m$

21. Solve the equation $v^2 = v_0^2 + 2ad$ for d.

▶ **22.** Solve each of these equations for a.

a. $v = v_0 + at$ **c.** $v^2 = v_0^2 + 2ay$

b. $y = v_0t + \frac{1}{2}at^2$ **d.** $v = \sqrt{2as}$

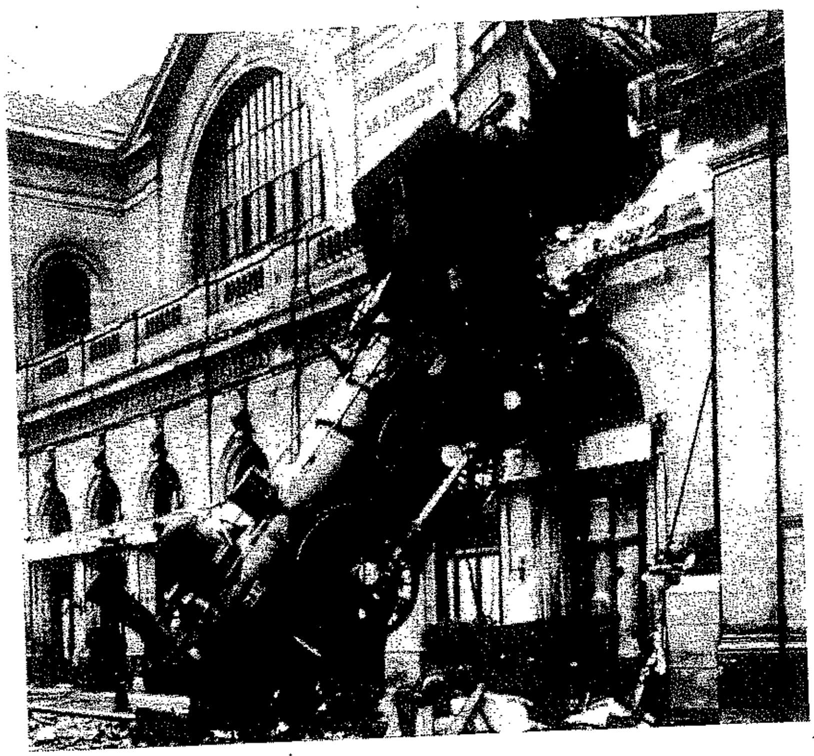


FIGURE 2–17. Did somebody miscalculate?

PHILOSOPHY CONNECTION

French philosopher and mathematician, Rene Descartes, (1596–1650), for whom the familiar Cartesian coordinates are named, is considered by many to have introduced analytical geometry as an appendix to his famous philosophical treatise on universal science, "Discourse on the Method of Rightly Conducting Reason and Seeking Truth in the Sciences". It has been noted that Descartes considered negative numbers "false", and avoided using them.

Units in Equations

All terms in an equation should have the same prefixes of the units.

Suppose you want to find the area of a wood plank. The plank is 15.0 cm wide and 2.50 m long. If you simply multiply length times width, you get 37.5 cm · m. This is not very useful. You should first make sure all terms in an equation have the same units. In this case, change the width to 0.150 m. You will then obtain an area of 0.375 m^2.

Always write the units with the values in an equation.

Most physical quantities have units as well as numerical values. When you substitute a value into an equation, you must write both the value and the unit. If your answer has the wrong units, you have made an error in your solution. When a term has several units, you can operate on the units like any other mathematical quantity.

Example Problem

Operating on Units

If $v = 11.0$ m/s and $t = 6.00$ s, find d using $d = vt$. Find the units for d.

Solution:

$d = vt = (11.0 \text{ m/s})(6.00 \text{ s}) = 66.0 \text{ (m/s) s} = 66.0 \text{ m}$

Units provide a useful way of checking the correctness of an equation.

Note that the units on the right, metres, are the units for distance. By inspecting the units, you will often be able to tell when you have set up the equation incorrectly.

Practice Problems

23. Identify the answers to these exercises using consistent units.
   a. Find the area of a rectangle 2 mm by 30 cm.
   b. Find the perimeter of a rectangle 25 cm by 2.00 m.

▶ **24.** Find which of the following equations are incorrect.
   a. area = (length)(width)(height) **c.** distance = (speed)$(\text{time})^2$
   b. time = distance/speed

CONCEPT REVIEW

4.1 Write a sentence that gives the same information as the equation $P = 4C$, where P represents the number of people and C represents the number of cars.

4.2 Write a sentence giving the same information as the equation $C = 2\pi r$, where C is the circumference of a circle and r is its radius.

4.3 Write an equation using the variables S and T to represent the following sentence: "There are twenty times as many students as teachers at this school."

4.4 **Critical Thinking:** The radius of Earth at the equator is 6378 km. Imagine a stiff wire wrapped around the equator of a perfectly smooth Earth. Suppose we now increased the length of the wire by 15 m, and shaped the wire into a circle centered at the center of Earth. How far above Earth's surface would the wire be? Explain your reasoning. If any information that you did not need was given in the problem, indicate it.

CHAPTER 2 REVIEW

SUMMARY

2.1 The Measure of Science

- The metre, second, and kilogram are the fundamental units of length, time, and mass in the SI system.
- Derived units are a combination of fundamental units.
- Large and small measurements, often used in physics, are most clearly written using scientific notation.
- Prefixes are used to change SI units by powers of 10.
- To be added or subtracted, quantities written in scientific notation must be raised to the same power of 10.
- Quantities written in scientific notation need not have the same power of 10 to be multiplied or divided.

2.2 Not All Is Certain

- All measurements are subject to some uncertainty.
- Precision is the degree of exactness with which a quantity is measured.
- Accuracy is the extent to which the measured and accepted values of a quantity agree.
- For a given measurement, the number of significant digits is limited by the precision of the measuring device.
- The last digit in a measurement is always an estimate. Only one estimated digit is significant.
- The result of any mathematical operation made with measurements can never be more precise than the least precise measurement.

2.3 Displaying Data

- Data are plotted in graphical form to show the relationship between two variables.
- The independent variable is the one the experimenter changes. It is plotted on the x- or horizontal axis. The dependent variable, which changes as a result of the changes made by the experimenter, is plotted on the y- or vertical axis.
- A graph in which data lie in a straight line is a graph of a linear relationship.
- A linear relationship can be represented by the equation $y = mx + b$.
- The slope, m, of a straight-line graph is the vertical change (rise) divided by the horizontal change (run).
- The graph of a quadratic relationship is a parabolic curve. It represents an equation $y = kx^2$.
- The graph of an inverse relationship between x and y is a hyperbolic curve. It represents an equation $xy = k$.

2.4 Manipulating Equations

- When solving an equation for a quantity, you should add, subtract, multiply, or divide in order to put that quantity alone on the left side of the equation.
- Units must be included when solving problems. The units must be the same on both sides of the equation. If this is not true, the equation is wrong.

KEY TERMS

metric system
SI
base units
second
metre
kilogram
derived units
scientific notation
prefixes
parallax
precision
accuracy
significant digits
independent variable
dependent variables
linear relationship
slope
y-intercept
quadratic relationship
inverse relationship

REVIEWING CONCEPTS

1. Why is the SI important?
2. List the common base units in the SI system.
3. How are the base units and derived units related?

4. Give the proper name for each multiple of the metre listed.
   a. 1/100 m b. 1/1000 m c. 1000 m
5. What determines the precision of a measurement?
6. Give an example of a measurement that is
   a. accurate but not precise.
   b. precise but not accurate.
7. How does the last digit differ from the other digits in a measurement?
8. Rick recorded a measurement as 76 000 nm.
   a. Why is it difficult to tell how many significant digits there are?
   b. How can the number of significant digits in such a number be made clear?
9. How do you find the slope of a straight-line or linear graph.
10. A person who has consumed alcohol usually has longer reaction times than a person who has not. Thus, the time between seeing a stoplight and hitting the brakes would be longer for the drinker than for the nondrinker.
    a. For a fixed speed, would the "reaction distance" for such a driver be longer or shorter than for a nondrinking driver?
    b. Would the slope of the graph of reaction distance versus speed have a larger or smaller slope?
11. During a laboratory experiment, the temperature of the gas in a balloon is varied and the volume of the balloon is measured. Which quantity is the independent variable? Which quantity is the dependent variable?
12. When plotting a graph of the experiment in the previous question,
    a. what quantity is plotted as the abscissa (horizontally)?
    b. what quantity is plotted as the ordinate (vertically)?
13. A relationship between the independent variable x and the dependent variable y can be written using the equation $y = ax^2$, where a is a constant.
    a. What is the shape of the graph of this equation?
    b. If $z = x^2$, what would be the shape of the graph $y = az$?
14. According to the formula $F = mv^2/r$, what relationship exists between
    a. F and r?
    b. F and m?
    c. F and v?
15. For the previous question, what type of graph would be drawn for
    a. F versus r?
    b. F versus m?
    c. F versus v?
16. How may units be used to check if an equation is written correctly?

APPLYING CONCEPTS

1. The density of a substance is the mass per unit volume of that substance.
   a. List a possible metric unit for density.
   b. Does density have a fundamental or derived unit?
   c. What is the SI unit for density?
2. Locate the size of the following objects in Figure 2–4.
   a. The width of your thumb.
   b. The thickness of a page in this book.
   c. The height of your classroom.
   d. The distance from your home to school.
3. Make a chart of sizes of objects similar to Figure 2–4. Include only objects you have measured. Some should be less than one millimetre, others several kilometres.
4. Make a chart of time intervals similar to Figure 2–4. Include intervals like the time between heartbeats, the time between provincial elections, the average lifetime of a human, the age of Canada, and so forth. Find as many very short and very long examples as you can.
5. Two students use a metre stick to measure the width of a lab table. One records an answer of 84 cm and the other 83.78 cm. Explain why neither answer is recorded correctly.
6. Suppose Madelaine measures the speed of light to be 2.999×10^8 m/s with an uncertainty of 0.006×10^8 m/s.
   a. Is this measurement more or less precise than the example on page 21?
   b. Is it more or less accurate?
7. Why can quantities with different units never be added or subtracted but can be multiplied or divided? Give examples to support your answer.

8. Suppose you receive \$5.00 at the beginning of a week and spend \$1.00 each day for lunch. You prepare a graph of the amount you have left at the end of each day. Would the slope of this graph be positive, zero, or negative? Why?
9. Data are plotted on a graph and the value on the *y*-axis is the same for each value of the independent variable. What is the slope? Why?
10. The graph of braking distance versus car speed is part of a parabola. Thus we write the equation $d = kv^2$. The distance, *d*, has units, metres, and velocity, *v*, has units metres/second. How could you find the units of *k*? What would they be?
11. Think of a relationship between two variables. In baseball you might consider the relationship between the distance the ball is hit and the speed of the pitch. Determine which is the independent variable and which is the dependent variable. In this example, the speed of the pitch is the independent variable. Choose your own relationship. If you can, think of other possible independent variables for the same dependent variable.
12. Aristotle said that the quickness of a falling object varies inversely with the density of the medium in which it is falling.
    a. According to Aristotle, would a rock fall faster in water (density 1000 kg/m^3), or in air (density 1 kg/m^3)?
    b. How fast would a rock fall in a vacuum? Based on this, why did Aristotle say that there could be no such thing as a vacuum?

PROBLEMS

2.1 The Measure of Science

1. Express the following numbers in scientific notation.
   a. 5 000 000 000 000 000 000 000 000 m
   b. 0.000 000 000 000 000 000 166 m
   c. 2 033 000 000 m
   d. 0.000 000 103 0 m
2. Convert each of the following measurements into meters.
   a. 42.3 cm
   b. 6.2 pm
   c. 21 km
   d. 0.023 mm
   e. 214 µm
   f. 570 nm
3. Rank the following mass measurements from smallest to largest: 11.6 mg, 1021 µg, 0.000 006 kg, 0.31 mg.
4. Add or subtract as indicated.
   a. 5.80×10^9 s + 3.20×10^8 s
   b. 4.87×10^{-6} m − 1.93×10^{-6} m
   c. 3.14×10^{-5} kg + 9.36×10^{-5} kg
   d. 8.12×10^7 g − 6.20×10^6 g

2.2 Not All Is Certain

5. State the number of significant digits in the following measurements.
   a. 248 m
   b. 0.000 03 m
   c. 64.01 m
   d. 80.001 m
6. State the number of significant digits in the following measurements.
   a. 2.40×10^6 kg
   b. 6×10^8 kg
   c. 4.07×10^{16} m
7. A consumer product manufacturer in the United States still uses Imperial units on labels, but also includes the metric equivalents. Examples are: 12 fluid ounces (9345.66 mL), 353 ft (107.59 m), 2.0 inch (50.80 mm). Report each metric equivalent using the correct number of significant digits.
8. Add or subtract as indicated and state the answer with the correct number of significant digits.
   a. 16.2 m + 5.008 m + 13.48 m
   b. 5.006 m + 12.0077 m + 8.0084 m
   c. 78.05 cm^2 − 32.046 cm^2
   d. 15.07 kg − 12.0 kg
9. Multiply or divide as indicated watching significant digits.
   a. $(6.2 \times 10^{18}\ m)(4.7 \times 10^{-10}\ m)$
   b. $(5.6 \times 10^{-7}\ m) \div (2.8 \times 10^{-12}\ s)$
   c. $(8.1 \times 10^{-4}\ km)(1.6 \times 10^{-3}\ km)$
   d. $(6.5 \times 10^5\ kg) \div (3.4 \times 10^3\ m^3)$
10. Tom did the following problems on his calculator, reporting the results shown. Give the answer to each using the correct number of significant digits.
    a. 5.32 mm + 2.1 mm = 7.4200000 mm
    b. 13.597 m × 3.65 m = 49.6290500 m^2
    c. 83.2 kg − 12.804 kg = 70.3960000 kg
11. A rectangular floor has a length of 15.72 m and a width of 4.40 m. Calculate the area of the floor to the best possible value using these measurements.

12. A yard is 33.21 m long and 17.6 m wide.
   a. What length of fence must be purchased to enclose the entire yard?
   b. What area must be covered if the yard is to be fertilized?
13. The length of a room is 16.40 m, its width is 4.5 m, and its height is 3.26 m. What volume does the room enclose?
14. The sides of a quadrangular plot of land are 132.68 m, 48.3 m, 132.736 m, and 48.37 m. What is the perimeter of the plot as can best be determined from these measurements?
15. A water tank has a mass of 3.64 kg when empty and a mass of 51.8 kg when filled to a certain level. What is the mass of the water in the tank?

2.3 Displaying Data

▶ 16. Figure 2–18 shows the mass of three substances for volumes between 0 and 60 cm^3.
   a. What is the mass of 30 cm^3 of each substance?
   b. If you had 100 g of each substance, what would their volumes be?
   c. Describe the meaning of the steepness of the lines in this graph (a single word is not a sufficient answer!).

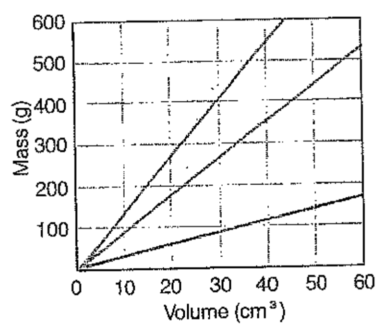


FIGURE 2-18. Use with Problem 16.

17. During an experiment, a student measured the mass of 10.0 cm^3 of ethanol. The student then measured the mass of 20.0 cm^3 of ethanol. In this way the data in Table 2–3 were collected.

Table 2–3

Volume (cm^3)	Mass (g)
10.0	7.9
20.0	15.8
30.0	23.7
40.0	31.6
50.0	39.6

   a. Plot the values given in the table and draw the curve that best fits all points.
   b. Describe the resulting curve.
   c. Use the graph to write an equation relating the volume to the mass of the alcohol.
   d. Find the units of the slope of the graph. What is the name given to this quantity?

▶ 18. During a class demonstration, an instructor placed a 1.0-kg mass on a horizontal table that was nearly frictionless. The instructor then applied various horizontal forces to the mass and measured the rate at which the mass gained speed (was accelerated) for each force applied. The results of the experiment are shown in Table 2–4.

Table 2–4

Force (N)	Acceleration (m/s^2)
5.0	4.9
10.0	9.8
15.0	15.2
20.0	20.1
25.0	25.0
30.0	29.9

   a. Plot the values given in the table and draw the curve that best fits all points.
   b. Describe, in words, the relationship between force and acceleration according to the graph.
   c. Write the equation relating the force and the acceleration that results from the graph.
   d. Find the units of the slope of the graph.

▶ 19. The teacher who performed the experiment in the previous problem then changed the procedure. The mass was varied while the force was kept constant. The acceleration of each mass was then recorded. The results are shown in Table 2–5.

Table 2–5

Mass (kg)	Acceleration (m/s^2)
1.0	12.0
2.0	5.9
3.0	4.1
4.0	3.0
5.0	2.5
6.0	2.0

a. Plot the values given in the table and draw the curve that best fits all points.
b. Describe the resulting curve.
c. According to the graph, what is the relationship between mass and the acceleration produced by a constant force?
d. Write the equation relating acceleration to mass given by the data in the graph.
e. Find the units of the constant in the equation.

2.4 Manipulating Equations

20. Each cubic centimetre of gold has a mass of 19.3 g. A cube of gold measures 4.23 cm on each edge.
  a. What is the volume of the cube?
  b. What is its mass?
21. Solve the equation

$$T = 2\pi\sqrt{l/g}$$

  a. for l.
  b. for g.
22. Each cubic centimetre of silver has a mass of 10.5 g.
  a. What is the mass of 65.0 cm^3 of silver?
  b. When placed on a beam balance, the 65.0-cm^3 piece of silver has a mass of only 616 g. What volume of the piece is hollow?
23. Assume that a small sugar cube has sides 1 cm long. If you had a box containing 1 mol of sugar cubes and lined them up side by side, how long would the line be? 1 mol = 6.02×10^{23} units.
24. The average distance between Earth and the sun is 1.50×10^8 km.
  a. Calculate the average speed, in km/h, of Earth assuming a circular path about the sun. Use the equation $v = \frac{2\pi r}{T}$.
  b. Convert your answer from km/h to m/s. Show all units.
25. ▶ The radius of Earth is 6.37×10^3 km.
  a. Find the speed, in km/h, resulting from the rotation of Earth, of a person standing on the equator.
  b. Convert your answer to m/s.
26. ▶ A child rides a merry-go-round horse that is 5.4 m from the centre. The rides lasts 10 min. During this time, the ride makes 24 revolutions. Find the speed of the child in metres per second. Use the equation $v = \frac{2\pi r}{T}$.
27. Manipulate the equation $v = d/t$ and find the answers to these problems using consistent units.
  a. Find the distance a bike travels in 1.5 min, if it is travelling at a constant speed of 20 km/h.
  b. How long will it take a car to travel 6000 m if its speed is a constant 30 km/h?
28. Water drips from a faucet into a flask at the rate of two drops every 3 s. One cubic centimetre (cm^3) contains 20 drops. What volume of water, in cubic decimetres (dm^3), will be collected in 1 h?
29. ▶ Tony's Pizza Shop ordered new 23-cm pizza pans. By mistake, 26-cm pans were delivered. Tony says that the difference is too small to worry about. As Tony's accountant, what would you say knowing materials cost about \$0.25/cm^2?

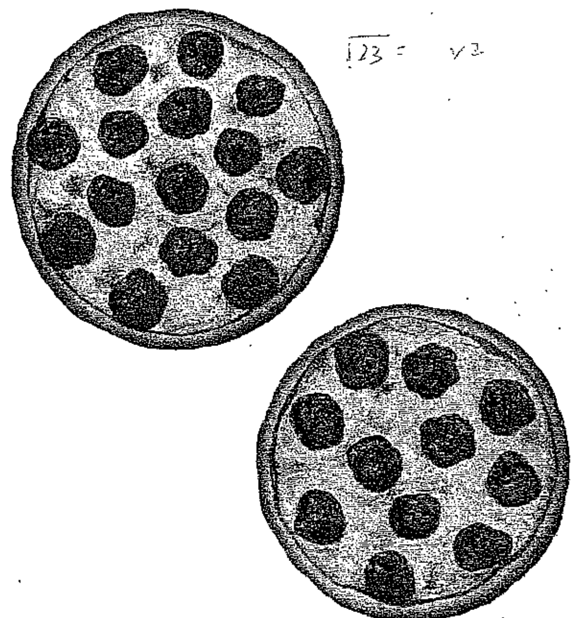


FIGURE 2–19.

THINKING PHYSIC-LY

Three students decide to buy a \$20.51 gift for a friend. Sally divides \$20.51 by 3 on her calculator and gets 6.83666667 as a result. Is this a reasonable amount for each student to pay? Explain.