Lesson Video: Physics 12 – Analysis of Systems of Masses - *free body diagrams* and *Newton's Second Law*

Lesson video: https://www.loom.com/share/2803644ca7fa4b22ae0978fc41e533f4

Dynamics

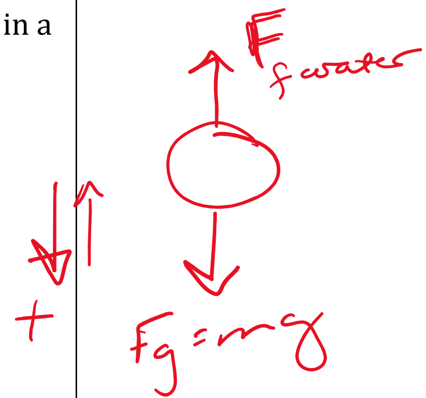
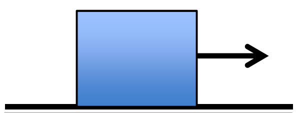
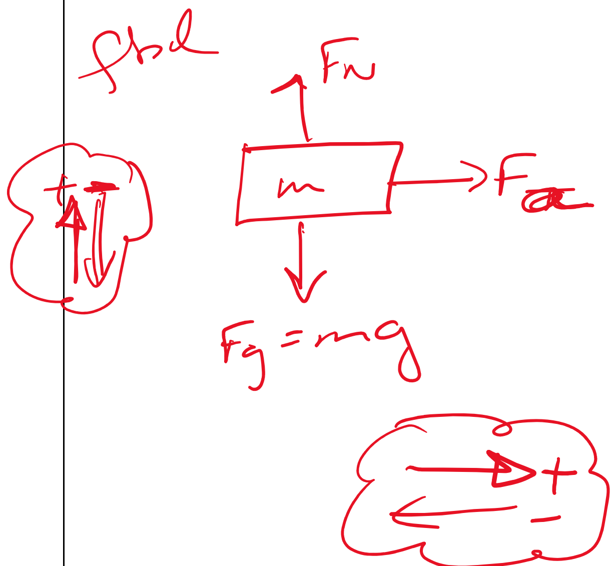
Refer to the example situations depicted below. For each situation, you will:

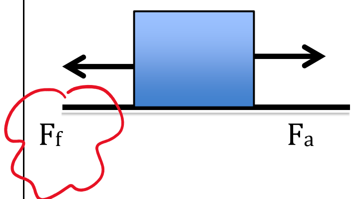
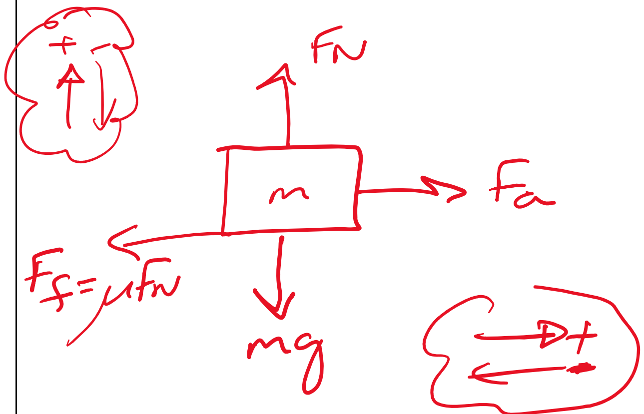
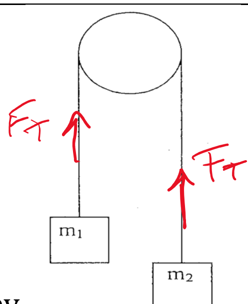
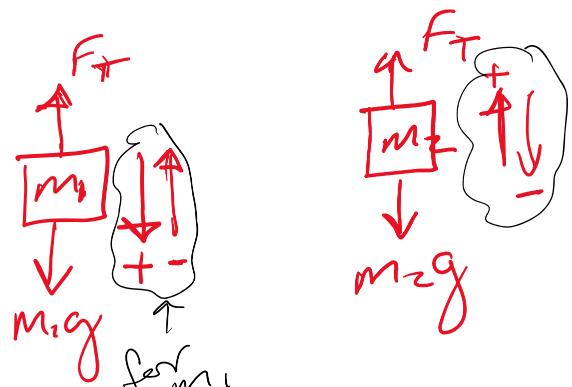
i) Draw and label a free body diagram

ii) Develop the equation for the acceleration of the mass or system of masses.

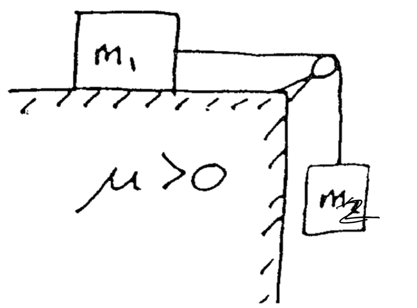
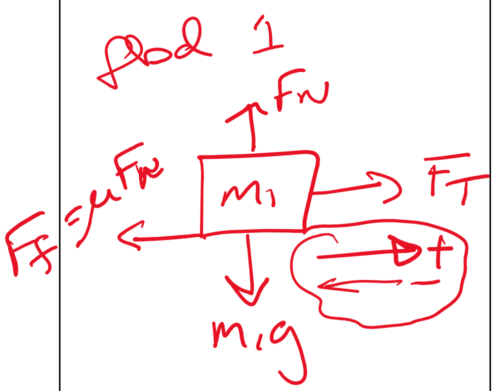
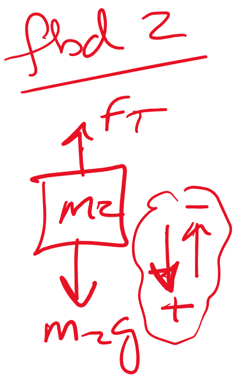
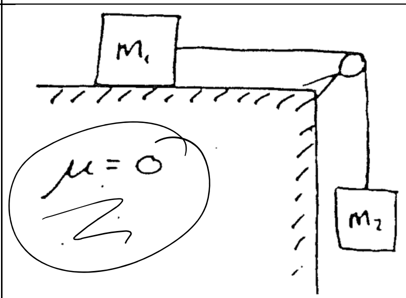
Analyze the example situations one at a time, in order.

	Example Situation	fbd (free body diagram) –	System Analysis using Newton's 2nd Law
1	Object in free fall (no friction or air resistance)	$F_g = mg$ (refer to fbd)	Equation for acceleration $\Sigma F = ma = F_g$ $m\vec{a} = mg$ $\therefore \vec{a} = g$
2	Object in free fall, but air resistance is ***not*** equal to zero $v_{terminal}$ = max const speed	F_{air} $F_g = mg$ (refer to fbd) $\Sigma F = 0$ if $\Sigma F = 0$ then $\vec{a} = 0$	Equation for acceleration $\Sigma F = ma = mg - F_{air}$ $\vec{a} = \frac{mg - F_{air}}{m}$ Condition for equilibrium (acceleration = zero, speed = "terminal velocity") $ma = mg - F_{air}$ ($a = 0$) $\therefore F_{air} = mg$

3	A stone falling through water (e.g. it has been dropped in a lake)	 	Equation for acceleration $\sum F = m\vec{a} = mg - F_{f\,water}$ $\vec{a} = \frac{mg - F_{f\,water}}{m}$ Condition for equilibrium $0 = mg - F_{f\,water}$ $\therefore F_{f\,water} = mg$
4	Mass = m Applied Force = F_a to the right No friction: F_f = 0N  not an fbd	 	Equation for acceleration $\sum F_y = ma_y = F_N - mg$ $a_y = 0$ $\quad 0 = F_N - mg$ $\therefore F_N = mg$ $\sum F_x = ma_x = F_a$ $a_x = \frac{F_a}{m}$

5	Mass = m Applied Force = F_a to the right Friction: $F_f \neq 0N$, to the left 		Equation for acceleration Refer to #4 for proof → $F_N = mg$ $\Sigma F_x = ma_x = F_a - F_f$ $ma_x = F_a - \mu F_N$ $ma_x = F_a - \mu mg$ $a_x = \frac{F_a - \mu mg}{m}$ Condition for equilibrium $\Sigma F = 0 \therefore a = 0$ $\Sigma F_x = 0 = F_a - \mu mg$ $\therefore F_a = \mu mg$
6	 Pulley supporting two hanging masses $F_T = F_T$ magnitude of F_T is equal everywhere	fbd for each mass 	Equation for acceleration $\Sigma F_1 = m_1 a = m_1 g - F_T$ $\Sigma F_2 = m_2 a = F_T - m_2 g$ $m_1 a + m_2 a = m_1 g - F_T + F_T - m_2 g$ $a(m_1 + m_2) = g(m_1 - m_2)$ $a = \frac{g(m_1 - m_2)}{m_1 + m_2}$ Equation for tension of the connecting cord $F_T = m_1 g - m_1 a$ $F_T = m_1(g - a)$ refer to answer key

Sign convention must be consistent with respect direction of acceleration. If m_1 is accelerating down then m_2 is up.

7	 	 $F_N = m_1 g$	Equation for acceleration $\Sigma F_1 = m_1 a = F_T - \mu m_1 g$ $\Sigma F_2 = m_2 a = m_2 g - F_T$ $m_1 a + m_2 a = m_2 g - \mu m_1 g$ $a = \frac{g(m_2 - \mu m_1)}{m_1 + m_2}$ Equation for tension of the connecting cord $F_T = m_2(g - a)$ $F_T = m_1 a + \mu m_1 g$ $F_T = m_1(a + \mu g)$ Condition for equilibrium $\vec{a} = 0$ $0 = \frac{g(m_2 - \mu m_1)}{m_1 + m_2}$ $\therefore m_2 - \mu m_1 = 0$ $\therefore m_2 = \mu m_1$ $\mu = \frac{m_2}{m_1}$
8		$a = \frac{g(m_2 - \mu m_1)}{m_1 + m_2}$ $\Sigma F = ma$ $(m_1 + m_2)a = m_2 g$ $a = \frac{m_2 g}{m_1 + m_2}$	Equation for acceleration $a = \frac{m_2 g}{m_1 + m_2}$ Equation for tension of the connecting cord $F_T = m_2(g - a)$ $F_T = m_1 a$

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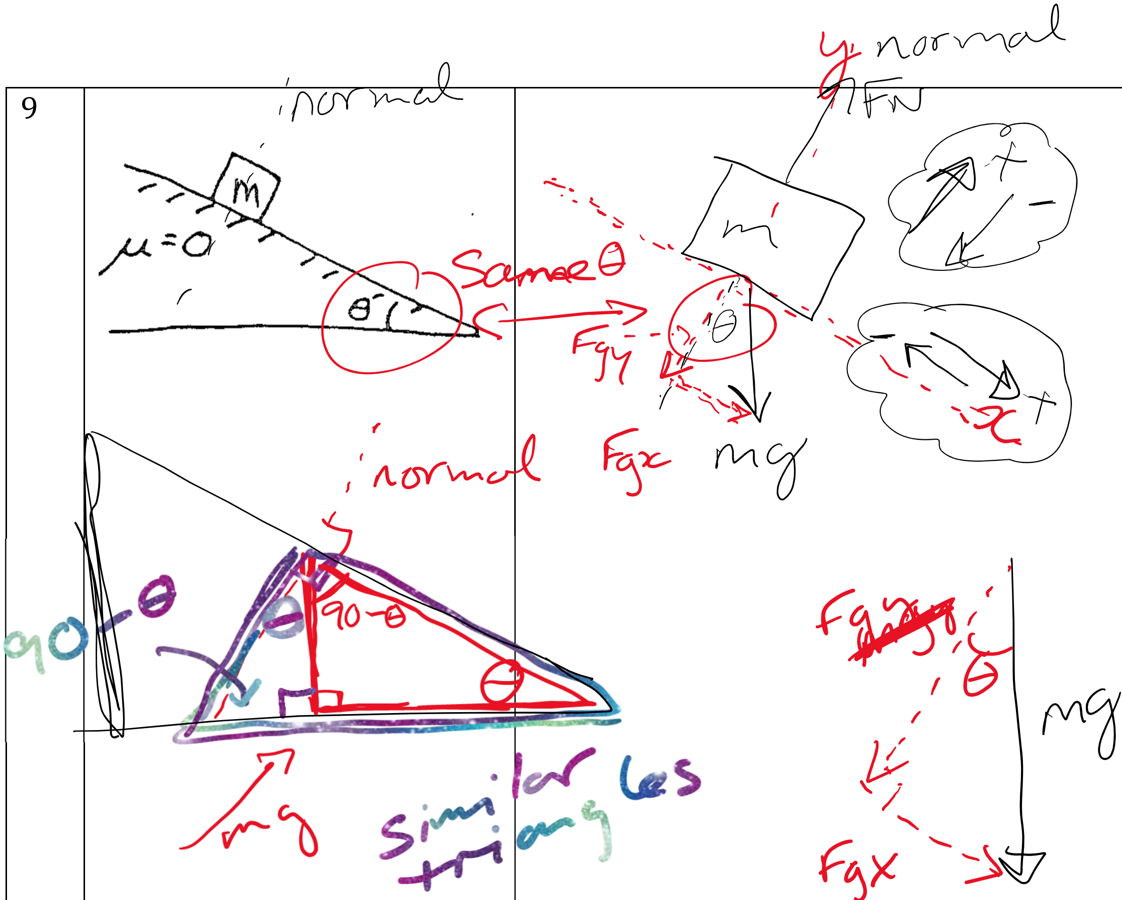


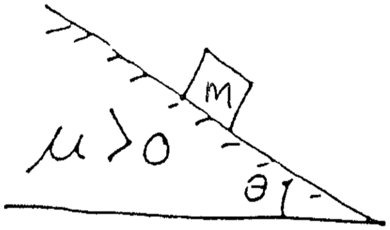
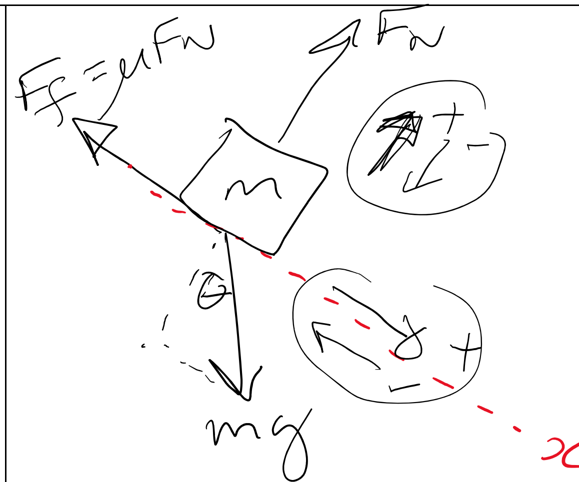
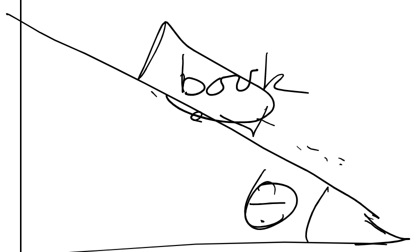
Equation for acceleration

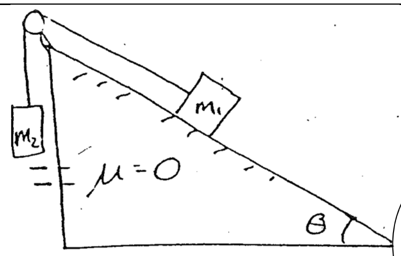
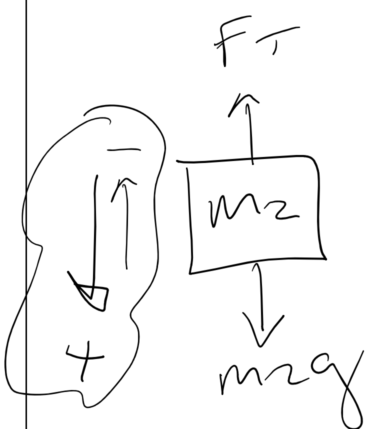
$$\sum F_y = ma_y = F_N - mg\cos\theta$$

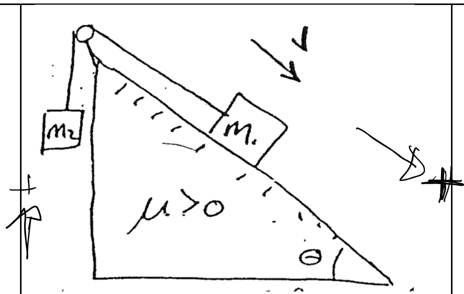
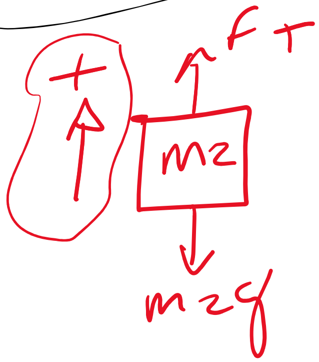
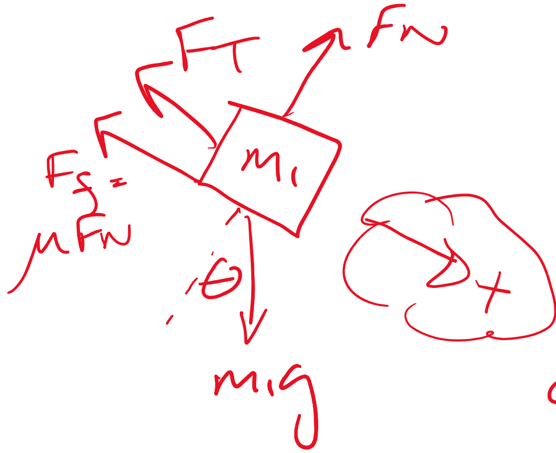
$$a_y = 0 \qquad \therefore F_N = mg\cos\theta$$

$$\sum F_x = ma_x = mg\sin\theta$$

$$\therefore \vec{a}_x = g\sin\theta$$

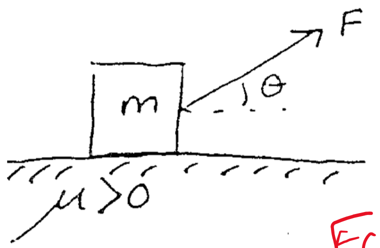
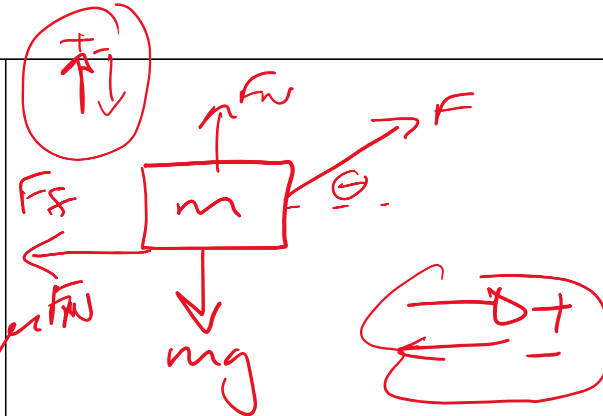
10	 	  	Equation for acceleration $F_N = mg\cos\theta$ (refer to #9 for proof) $\sum F_x = ma_x = mg\sin\theta - \mu mg\cos\theta$ $\vec{a} = g(\sin\theta - \mu\cos\theta)$ Condition for equilibrium $\vec{a} = 0$ $\frac{0}{g} = \frac{g}{g}(\sin\theta - \mu\cos\theta)$ $0 = \sin\theta - \mu\cos\theta$ $\mu = \frac{\sin\theta}{\cos\theta} = \tan\theta$ $\therefore \mu = \tan\theta$

11	 		Equation for acceleration $\Sigma F_1 = m_1 a = F_T - m_1 g \sin\theta$ $\Sigma F_2 = m_2 a = m_2 g - F_T$ $(m_1 + m_2) a = m_2 g - m_1 g \sin\theta$ $a = g\frac{(m_2 - m_1 \sin\theta)}{m_1 + m_2}$ Equation for tension of the connecting cord $F_T = m_2 (g - a)$ $F_T = m_1 (a + g \sin\theta)$ Condition for equilibrium $a = 0$ ∴ $m_2 - m_1 \sin\theta = 0$ ∴ $m_2 = m_1 \sin\theta$

12	 *The system is sliding to the right (m_1 is sliding down the slope, and m_2 is moving upward)* 	 $F_N = mg\cos\theta$	Equation for acceleration $\Sigma F_2 = m_2 a = F_T - m_2 g$ $\Sigma F_1 = m_1 a = m_1 g\sin\theta - F_T - \mu m_1 g\cos\theta$ $a(m_1+m_2) = m_1 g\sin\theta - m_2 g - \mu m_1 g\cos\theta$ $a = \frac{m_1 g\sin\theta - m_2 g - \mu m_1 g\cos\theta}{m_1+m_2}$ Equation for tension of the connecting cord $F_T = m_2(a+g)$ $F_T = m_1 g\sin\theta - \mu m_1 g\cos\theta - mg$ Condition for equilibrium $a = 0$ $0 = m_1 g\sin\theta - m_2 g - \mu m_1 g\cos\theta$ $m_1\sin\theta = m_2 + \mu m_1\cos\theta$

if $m_1 g\sin\theta > m_2 g$ then F_f is to left (up slope)

if $m_1 g\sin\theta < m_2 g$ then F_f is to the right (down slope)

13	 $a_y = 0$	 $F_f = \mu F_N$	Equation for acceleration $\Sigma F_x = ma_x = F\cos\theta - F_f$ $ma_x = F\cos\theta - \mu F_N$ $ma_x = F\cos\theta - \mu(mg - F\sin\theta)$ $a_x = \frac{F\cos\theta - \mu mg + \mu F\sin\theta}{m}$ Condition for equilibrium $0 = F\cos\theta - \mu mg + \mu F\sin\theta$

$$\Sigma F_y = 0 = F_N + F\sin\theta - mg$$

$$\therefore F_N = mg - F\sin\theta$$