Projectiles in 2-D Lesson – Textbook reference: Chapter 3.1 to 3.3

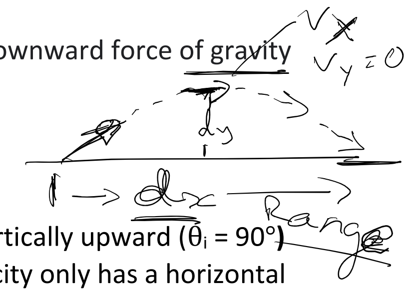
Lesson video: https://www.loom.com/share/1b1d857b281b4953a77c031f51efd469

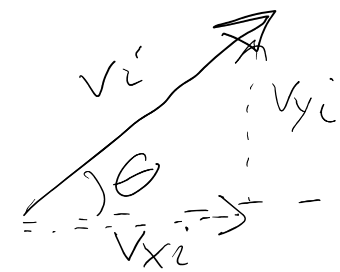
Vocabulary and key concepts:

- **Projectile** = an object that is projected (launched) or dropped and continues in motion influenced only by the downward force of gravity
- **Trajectory** = path of the projectile (in free fall, the shape of the trajectory is parabolic)
- **x component** = horizontal component
- **y component** = vertical component
- **Range** = maximum horizonal displacement of the projectile (Δd_x)
- **Maximum height (d_y)** occurs when v_y = 0m/s, ***BUT*** v_x is NOT equal to zero unless the projectile was launched vertically upward (θ_i = 90°)
- **Minimum speed** occurs at maximum height, when v_y = 0m/s. In this case, if v_y = 0m/s, then speed = v_x (the velocity only has a horizontal component at that instant)
- In free-fall, $\boldsymbol{\sum F_y = F_g}$, therefore $\boldsymbol{ma_y = mg}$, therefore $\boldsymbol{a_y = g}$ **[toward the centre of the Earth]**
- In free-fall, $\boldsymbol{\sum F_x = 0N}$ therefore $\boldsymbol{ma_x = 0N}$, therefore $\boldsymbol{a_x = 0m/s^2}$, therefore $\boldsymbol{v_{xi} = v_{xf} = v_x}$

$\sum F_x = 0N$

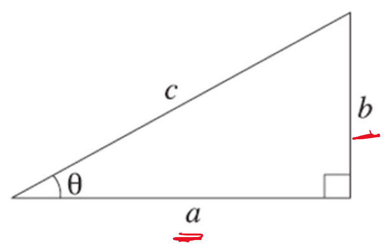
$\sum F_y = F_g$





Mathematical tools needed: *Pythagorean Theorem and Trigonometry*

For Right-angled Triangles:



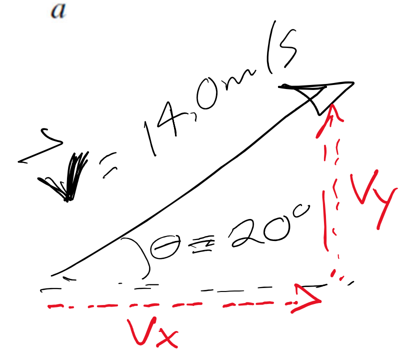
$$a^2 + b^2 = c^2$$

$$\sin\theta = \frac{b}{c} \qquad \cos\theta = \frac{a}{c} \qquad \tan\theta = \frac{b}{a}$$

$$\text{area} = \frac{1}{2}ab$$

Rearranging the sine and cosine relationships:

- ***b*** = ***c***×*sin*θ v_y
- v_x ***a*** = ***c***×*cos*θ



Example: velocity vector: ***v = 14.0 m/s [20.0° above the horizontal]***
Referring to the right-angled triangle shown above:

c = velocity vector = **v**	$\mathbf{v^2 = v_x^2 + v_y^2}$	
a = x component of velocity = $\mathbf{v_x}$	$\mathbf{v_x}$ = **v**cosθ	= (14.0m/s)×cos 20.0° = 13.2 m/s
b = y component of velocity = $\mathbf{v_y}$	$\mathbf{v_y}$ = **v**sinθ	= (14.0m/s)×sin20.0° = 4.79 m/s

$\therefore v = \sqrt{v_x^2 + v_y^2}$

Example 1: *Launch angle $\theta_i > 0°$ above the horizontal (but not equal to 90°)*

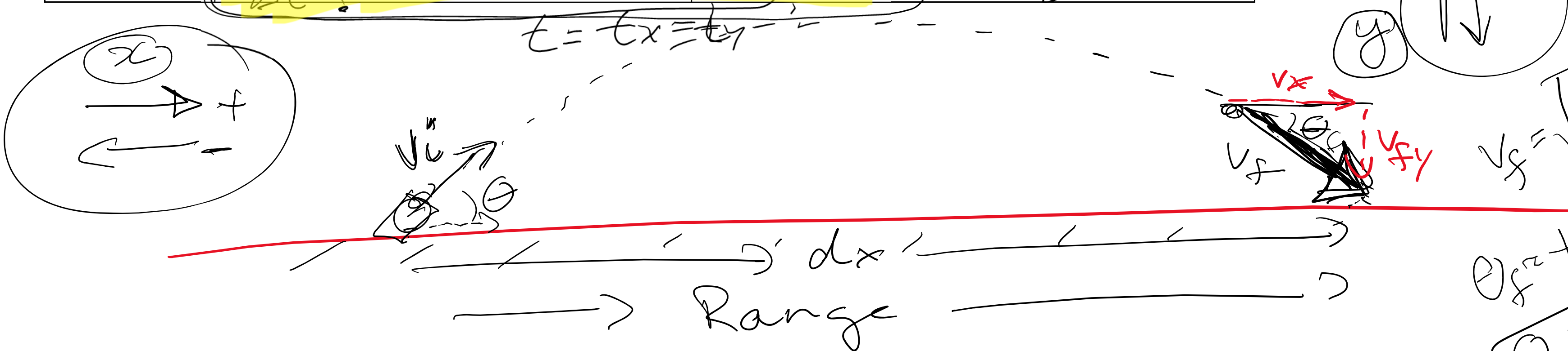
Case specific parameters:

- Projectile lands at the same level that it was launched from (Δd_y = 0m)
- Determine relationship for ***maximum height*** = H
- Determine relationship for ***minimum speed*** (= speed at the top of the trajectory)
- Determine relationship for ***Range*** = R = d_{xf}
- Determine final velocity: $\mathbf{v_f}$ = ***vector sum*** of $\mathbf{v_x} + \mathbf{v_{fy}}$

Given information:

Variable	x component (horizontal)	y component (vertical)
v_i	v_{ix} = $v_x = v_i \cos\theta$	v_{iy} = + $v_i \sin\theta$
v_f	v_{fx} =	v_{fy} = ?
Δd	Δd_x = ?	Δd_y = 0 m
a	a_x = 0 m/s²	a_y = − 9.80 m/s²
Δt	Δt = ?	Δt = ?

$t = t_x = t_y$



$v_f = \sqrt{v_x^2 + v_{fy}^2}$

$\theta_f = \tan^{-1}\left(\frac{v_{fy}}{v_x}\right)$

θ_f = ?

Max H

y

$v_{fy} = 0$ m/s

$a = -9.8$ m/s²

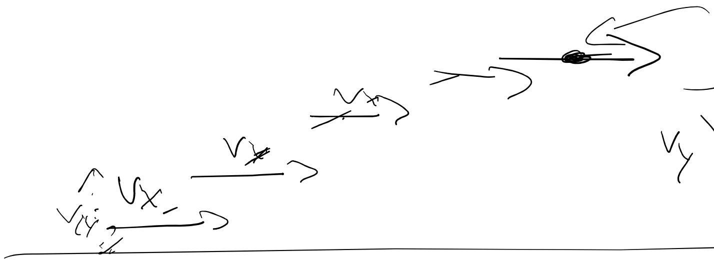
$v_{iy} = v_i \sin\theta$

Δd = ?

$$\boxed{v_{fy}^2 = 2a_y\Delta d_y + v_{iy}^2}$$

$$\Delta d_y = \frac{-v_{iy}^2}{2a_y} = \ ? = H$$

Minimum speed at top of trajectory



$V_y = 0$

$\boxed{V_x = V_i \cos\theta} = ?$

Range = $\boxed{d_x = V_x t}$

$a_x = 0$

$\therefore V_x = \frac{d_x}{t}$

$\boxed{d_x = V_i \cos\theta \times t}$

~~y~~ y whole trip

$V_{iy} = V_i \sin\theta$

$\Delta d_y = 0$

$a_y = -9.80\ m/s^2$

$t = ?$

$\cancel{V_f}$

$\frac{\Delta d_y}{t} = \frac{\frac{1}{2} a_y t^2 + V_{iy} t}{t}$

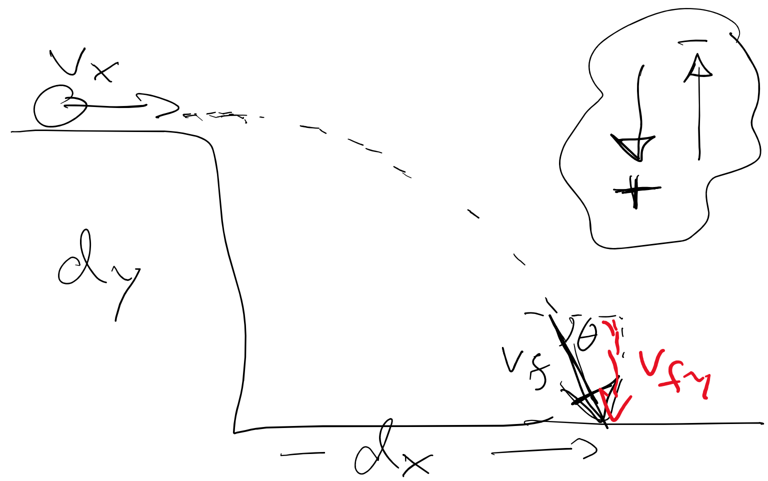
$0 = \frac{1}{2} a_y t + V_{iy}$

$t = \frac{-V_i \sin\theta \times 2}{-9.8}$

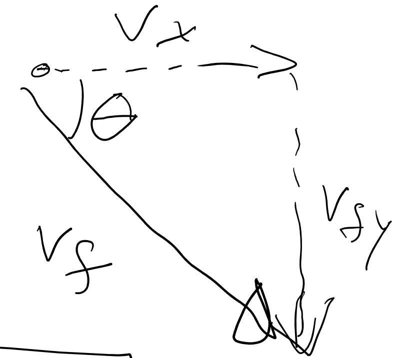
Example 2: *Launch angle $\theta_i = 0°$, along the horizontal* therefore $v_x = v_i$ and $v_{yo} = 0.0$ m/s

Given information:

Variable	x	y
v_i	$v_{ix} =$ v_x	$v_{iy} =$ 0 m/s
v_f	~~v_{fx}~~ $=$	$v_{fy} =$
Δd	$\Delta d_x =$ $v_x \cdot t = ?$	$\Delta d_y =$ d_y
a	$a_x =$ 0	$a_y =$ $+9.80$ m/s^2
Δt	$\Delta t =$	$\Delta t =$



find $\vec{v}_f$



$$v_f = \sqrt{v_x^2 + v_{fy}^2}$$

$$\theta = \tan^{-1}\left[\frac{v_{fy}}{v_x}\right] = ?$$

$$\boxed{v_f = \_\_\_\_\_\_ \text{ m/s } [\_\_\_\_\_°\text{ below the horizontal}]}$$

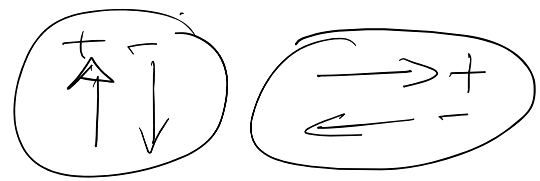
Example 3: ***Launch angle $\theta_i > 0°$ above the horizontal (but not equal to 90°)***

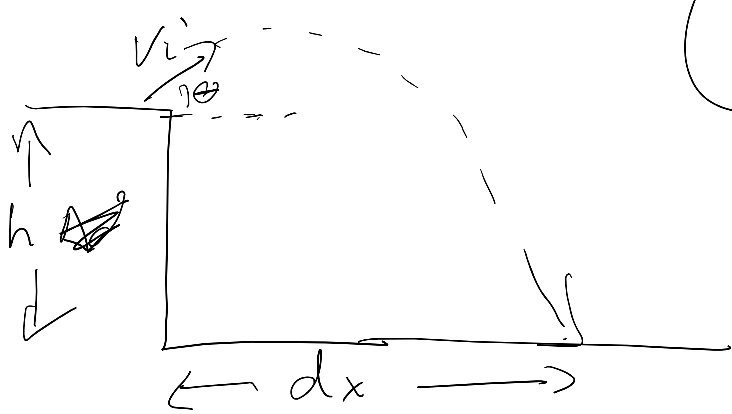
Case specific parameters:

- Projectile lands below the level that it was launched from (Δd_y is negative)

Given information:

Variable	x	y
v_i	$\cancel{v_{ix}} =$ $v_x = v_i \cos\theta$	$v_{iy} =$ $v_i \sin\theta$
v_f	$\cancel{v_{fx}} =$	$v_{fy} =$?
Δd	$\Delta d_x =$ $= v_x t$	$\Delta d_y =$ $-h$
a	$\cancel{a_x} =$ 0	$a_y =$ $-9.80\ m/s^2$
Δt	$\Delta t =$?	$\Delta t =$?





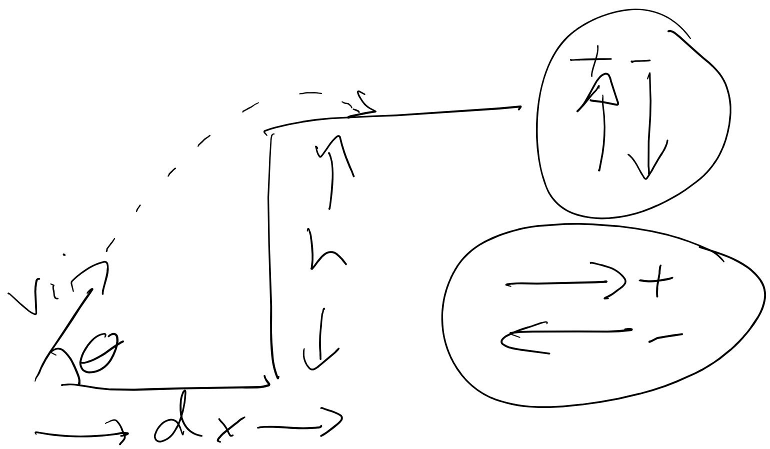
Example 4: ***Launch angle $\theta_i > 0°$ above the horizontal (but not equal to 90°)***

Case specific parameters:

- Projectile lands above the level that it was launched from (Δd_y is positive)

Given information:

Variable	x	y
v_i	~~v_{ix}~~ = $v_x = v_i \cos\theta_i$	$v_{iy} = v_i \sin\theta_i$
v_f	~~v_{fx}~~ =	v_{fy} = ?
Δd	$\Delta d_x = v_x t$	$\Delta d_y = +h$
a	$a_x = 0$	$a_y = -9.80 \text{ m/s}^2$
Δt	Δt = ?	Δt = ?



Example 5: ***Launch angle θ_i > 0° below the horizontal (but not equal to 90°)***

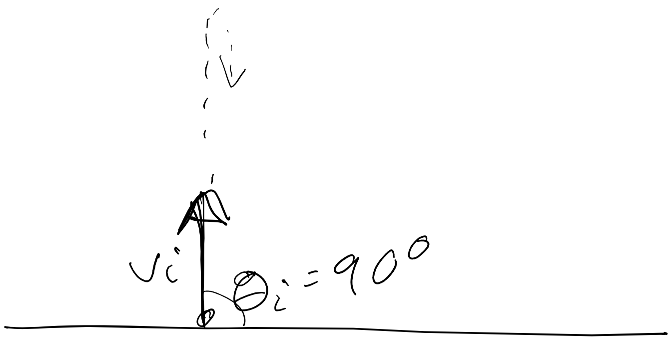
Given information:

Variable	x	y
v_i	$v_{ix} = v_x = v_i \cos\theta$	$v_{iy} = +v_i \sin\theta$
v_f	~~v_{fx}~~ $= v_x$	$v_{fy} =$?
Δd	$\Delta d_x = v_x t$	$\Delta d_y = +h$
a	$a_x = 0$	$a_y = +9.80 m/s^2$
Δt	$\Delta t =$?	$\Delta t =$?

Example 6: ***Launch angle θ_i equal to 90° (i.e. a 1-D projectile of the type you studied in Physics 11)***

Given information:

Variable	x	y
v_i	v_{ix} = $v_x = 0$ m/s	v_{iy} = v_i
v_f	v_{fx} =	v_{fy} = 0
Δd	Δd_x = 0	Δd_y = h
a	a_x = 0	a_y = -9.80 m/s²
Δt	Δt =	Δt = ?



$$V_x = V_i \cos \cancel{\theta}^{\,90°} = \underline{\underline{0}}$$