Lesson video #1 - Conservation of momentum theory, with examples

Link to the lesson video: https://www.loom.com/share/8740a63a7a764b349c4e163fc512c87b

The Law of Conservation of momentum = the total momentum of all objects within a *system* before an interaction (e.g. collision or explosion) = the total momentum of all the objects within the system after the interaction

Definition: "system": A "system" refers to all objects involved in an interaction

- For example:
  - when a ball bounces against a wall, the "system" is the ball and the wall
  - when a ball is bounced off the ground (Earth), the "system" is the ball and the Earth
  - when 2 cars crash into each other, the "system" is both cars
  - when a bomb explodes and breaks into many pieces, the "system" is the bomb (1 piece before the collision, and all the pieces after the collision)
  - when 2 ice skaters push again each other and slide away in opposite directions, the "system" is the 2 skaters
  - When a meteorite crashes into the Earth, the "system" is meteorite and the Earth

The Law of Conservation of Momentum can be understood by considering Newton's 3rd Law (action-reaction).

- Consider the system of 2 ice skaters pushing against each other
- Skater A experiences a force of F_{BA} (the force with which B pushes on A)
- Skater B experiences a force of F_{AB} (the force with which A pushes on B)
- Due to Newton's 3rd Law, we know that $F_{BA} = - F_{AB}$ (equal and opposite)
- So, although each individual object within the system experiences a non-zero net force, the net force of all objects within the system is zero.

$$F_{BA} = - F_{AB}$$

$$\Sigma F = m\vec{a} = m\frac{\Delta \vec{v}}{\Delta t}$$

So, $$m_A \Delta v_A / t = - m_B \Delta v_B / t$$

Since the two objects are part of a system, and the forces they exert are on against each other, the duration (time) is the same for both. So, time can be cancelled out of the equation.

$$m_A \Delta v_A = - m_B \Delta v_B$$

Expanding this equation: $$m_A(v_A' - v_A) = - m_B(v_B' - v_B)$$

Then, multiplying to removing the brackets: $$m_A v_A' - m_A v_A = - (m_B v_B' - m_B v_B)$$
$$m_A v_A' - m_A v_A = - m_B v_B' + m_B v_B$$

Then, since *mv = p* …. $$p_A' - p_A = - p_B' + p_B$$

Then, rearrange to gather the "before the interaction" momenta on one side of the equation, and "after the interaction" momenta on the other side:

$$\vec{p}_A + \vec{p}_B = \vec{p}_A' + \vec{p}_B'$$

So, this means, if p_T refers to the sum of the momenta of all objects in the system:

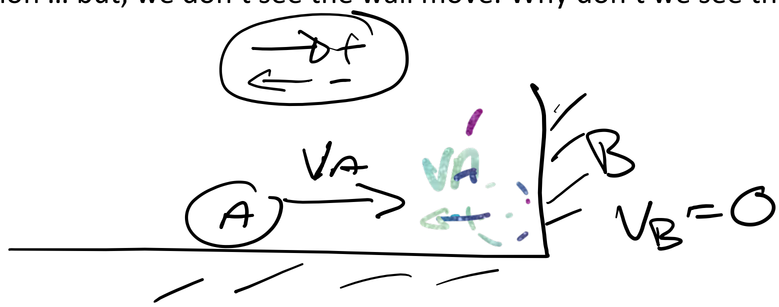
$$\vec{p}_T = \vec{p}_T'$$

Examples and applications:

1) A ball bounces off a wall: The ball is A, and the wall is B

$$\left.\begin{aligned} p_T &= p_T' \\ p_A + p_B &= p_A' + p_B' \\ m_A v_A + m_B v_B &= m_A v_A' + m_B v_B' \end{aligned}\right\} \text{cons. of } \vec{p}$$

Before the collision, $v_A > 0$ (positive direction), and $v_B = 0$
After the collision v_A is non-zero, in the negative direction, which means v_B must be greater than zero, in the positive direction … but, we don't see the wall move. Why don't we see the wall move?



$$m_A v_A + \cancel{m_B v_B}^{\,0} = m_A v_A' + m_B \underline{v_B'}$$

$$v_B' = \frac{m_A v_A - m_A v_A'}{m_B} = \frac{m_A(v_A - v_A')}{m_B}$$

wall

e.g. $v_A = 1.0\,m/s$
$v_A' = -1.0\,m/s$

$$v_B' = \frac{m_A(1\,m/s - (-1\,m/s))}{m_B}$$

$$v_B' = \frac{2 m_A}{m_B}$$

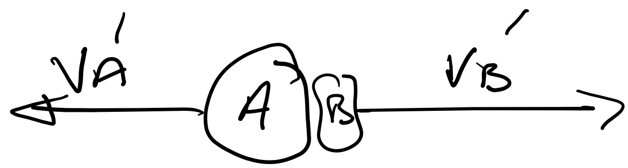
Wall (B) does move after the ball hits it. But m_B (wall) is huge $m_B >>>> m_A$ ∴ v_B' is VERY small

2) A bomb explodes into 2 pieces (not realistic, but the theory works in real life, where a bomb explodes into many pieces scattering in many directions … but, for Physics 11 we'll stick with 2 pieces, because then we can stick with 1-D math, rather than 2-D or 3-D).

$$\left.\begin{aligned} p_T &= p_T' \\ p_A + p_B &= p_A' + p_B' \\ m_A v_A + m_B v_B &= m_A v_A' + m_B v_B' \end{aligned}\right\} \text{cons. of } \vec{p}$$

Before the ~~collision~~ explosion, the bomb is at rest. So, $p_T = 0$.
Therefore, after the collision the vector sum of the momenta of the 2 scattered bomb fragments must be zero.

$\underline{p = 0}$ $\quad V = 0$



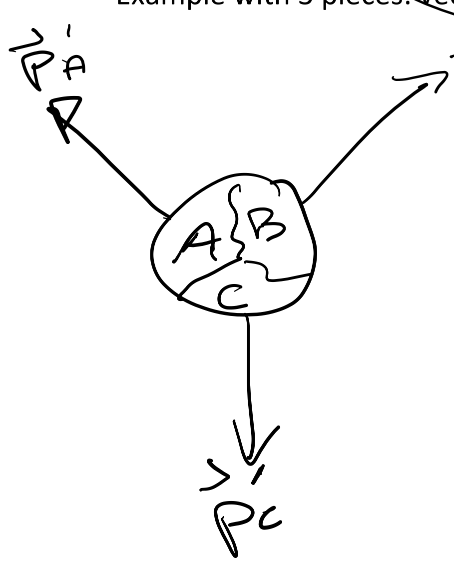
$$\vec{p}_T = \vec{p}_T$$

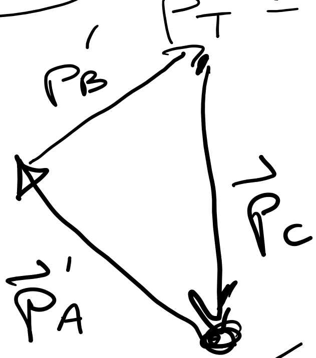
$$0 = m_B v_B' - m_A v_A'$$

$$m_A v_A' = \underline{m_B} v_B'$$

- Example with 3 pieces: vector diagram

$\vec{p}_T = 0 \qquad \vec{p}_T' = 0$





$\underline{\Sigma \vec{p} = 0}$

3) Car crash examples: Car A ($m_A = 1.50\times10^3$ kg), and Car B ($m_B = 9.80\times10^2$ kg).

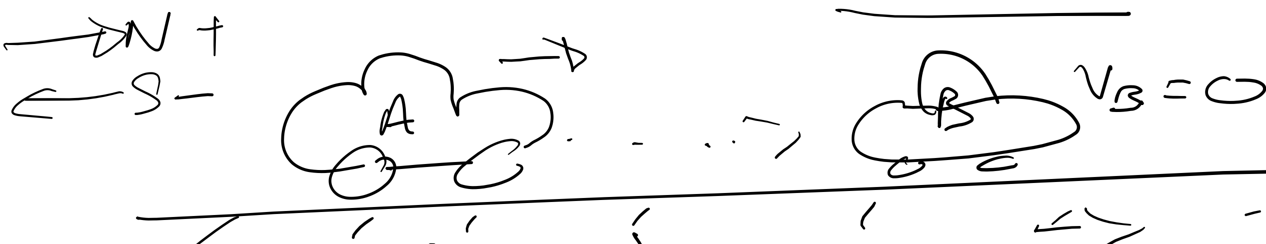
a. Car A is moving North at a speed of 20.0 m/s, and Car B is stationary. After the collision the 2 cars stick together. Determine the velocity of the 2 cars (together) after the collision.

$$\vec{p}_T = \vec{p}_T'$$

$$m_A\vec{v}_A + \cancel{m_B\overset{0}{\vec{v}_B}} = (m_A + m_B)\vec{v}'$$

$$\vec{v}' = \frac{m_A\vec{v}_A}{m_A + m_B} = \frac{(1500)(20)}{1500 + 980} = \boxed{12.1\text{ m/s [North]}}$$

b. Car A is moving North at a speed of 20.0 m/s, and Car B is stationary. After the collision Car B moves North at 15.0m/s. Determine the velocity of the Car A after the collision.



$$\vec{p}_T = \vec{p}_T'$$

$$m_A v_A + \cancel{m_B \overset{0}{v_B}} = m_A v_A' + m_B v_B'$$

$$v_A' = \frac{m_A v_A - m_B v_B'}{m_A}$$

$$= \frac{(1500)(20) - (980)(15)}{1500} = \boxed{\cancel{v_A' =}\ 10.2\text{ m/s [North]}}$$

c. Car A is moving North at a speed of 20.0 m/s, and Car B is moving South at 15.0 m/s. After the collision the 2 cars stick together. Determine the velocity of the 2 cars (together) after the collision.

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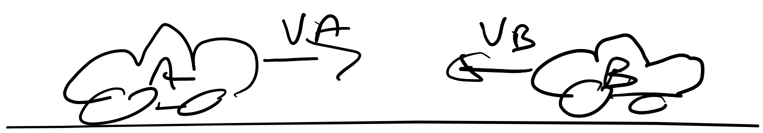


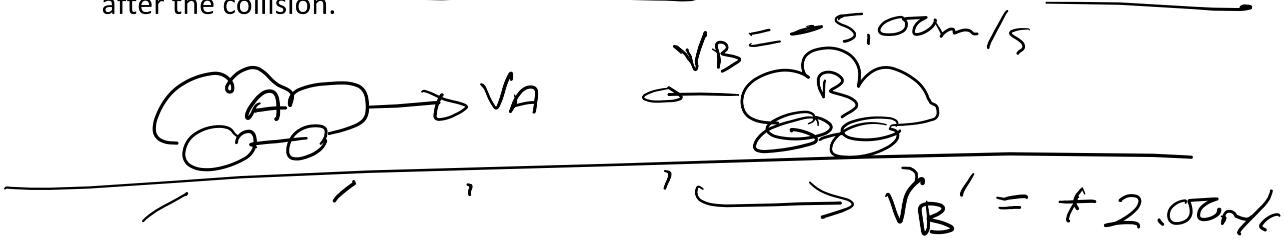
$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

$$= (m_A + m_B)\, v'$$

$$v' = \frac{m_A v_A + m_B v_B}{m_A + m_B} = \frac{(1500)(20) + (980)(-15)}{1500 + 980}$$

$$\boxed{v' = 6.17\ \text{m/s [North]}}$$

d. Car A is moving at 4.00 m/s [North], and Car B is moving at 5.00 m/s [South]. After the collision Car B rebounds ~~and moves~~ at 2.00 m/s [North]. Determine the velocity of Car A after the collision.



$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

$$v_A' = \frac{m_A v_A + m_B v_B - m_B v_B'}{m_A}$$

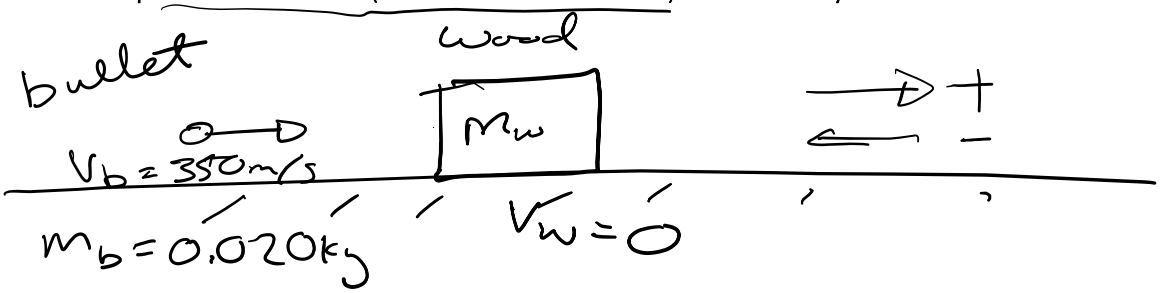
$$= \frac{(1500)(4) + (980)(-5) - (980)(2)}{1500}$$

$$v_A' = -0.57\ \text{m/s} \quad \therefore \boxed{v_A' = 0.57\ \text{m/s [South]}}$$

4) Bullet/block examples:

- mass of the bullet: m_b = 20.0 grams; speed of the bullet before collision = 350.0 m/s
- mass of the block of wood: m_W = 3.00 kg; speed of the block before collision v_W = 0.00 m/s

   a. After the bullet hits the block it becomes embedded within the block. Determine the speed of the block (with embedded bullet) immediately after the collision.



$$m_b V_b + \cancel{m_w V_w}^{\,0} = (m_b + m_w) V'$$

$$V' = \frac{m_b V_b}{m_b + m_w} = \frac{(0.02)(350)}{(0.02)+(3)}$$

$$\boxed{V' = 2.47 m/s}$$

   b. After the bullet hits the block it travels completely through the block and emerges with a speed of 150.0 m/s. Determine the speed of the block immediately after the collision.

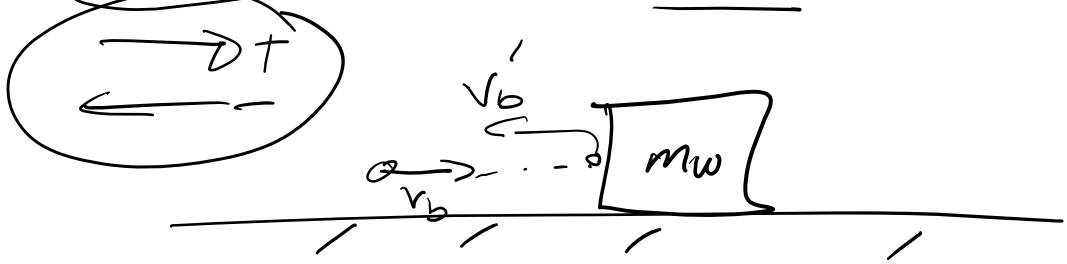
$V_w > 0$

$$m_b V_b + \cancel{m_w V_w}_{\,0} = m_b V_b' + m_w \underline{V_w'}$$

$$V_w' = \frac{m_b V_b - m_b V_b'}{m_w} = \frac{m_b (V_b - V_b')}{m_w}$$

$$= \frac{(0.02)(350-150)}{3} = \boxed{1.33 m/s}$$

c. After the bullet hits the block it bounces off the block and rebounds with a speed of −100.0 m/s. Determine the speed of the block immediately after the collision.



$$m_b V_b + m_w V_w = m_b V_b' + m_w V_w'$$

$$V_w' = \frac{m_b V_b + \overset{0}{\cancel{m_w V_w}} - m_b V_b'}{m_w}$$

$$= \frac{(0.02)(350) - (0.02)(-100)}{3}$$

$$\boxed{V_w' = 2.47\,m/s}$$