MATH SKILL HANDBOOK

FOR PHYSICS

I. Symbols

Δ	change in quantity	≡	is defined as
±	plus or minus a quantity	$a \times b$ ab $a(b)$	*a* multiplied by *b*
∝	is proportional to		
=	is equal to	$a \div b$ a/b $\frac{a}{b}$	*a* divided by *b*
≈	is approximately equal to		
≤	is less than or equal to		
≥	is greater than or equal to	$\sqrt{a}$	square root of *a*
<<	is much less than	$\|a\|$	absolute value of *a*
>>	is much greater than	$\log_b x$	log to the base *b* of *x*

II. Measurements and Significant Figures

Math is the language of physics. Physicists use mathematical equations to describe relationships among the measurements that they make. Each measurement is associated with a symbol that is used in physics equations. These symbols are called variables.

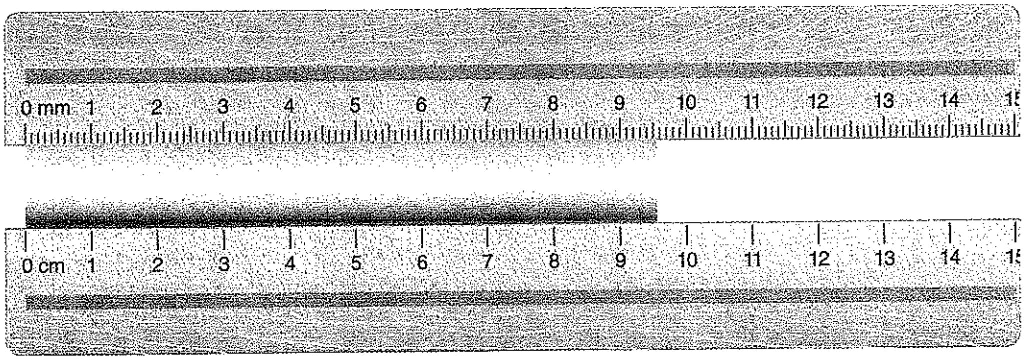
Significant Figures

All measured quantities are approximate and have significant figures. The number of significant figures indicates the precision of the measurement. Precision is a measure of exactness. The number of significant figures in a measurement depends on the smallest unit on the measuring tool. The digit farthest to the right in a measurement is estimated.

Example: In the figure below, what is the estimated digit for each of the measuring sticks used to measure the length of the rod?

By using the lower measuring tool, the length is between 9 and 10 cm. The measurement would be estimated to the nearest tenth of a centimeter. If the length was exactly on the 9-cm or 10-cm mark, record it as 9.0 cm or 10.0 cm.

By using the upper measuring tool, the length is between 9.5 and 9.6 cm. The measurement would be estimated to the nearest hundredth of a centimeter. If the length was exactly on the 9.5-cm or 9.6-cm mark, record it as 9.50 cm or 9.60 cm.



All nonzero digits in a measurement are significant figures. Some zeros are significant, and some are not. All digits between and including the first nonzero digit from the left through the significant figure on the right are significant. Use the following rules when determining the number of significant figures.

1. Nonzero digits are significant.
2. Final zeros after a decimal point are significant.
3. Zeros between two significant figures are significant.
4. Zeros used only as placeholders are not significant.

Example: State the number of significant figures in each measurement.

5.0 g has two significant figures.	*Using rules 1 and 2*
14.90 g has four significant figures.	*Using rules 1 and 2*
0.0 has one significant figure.	*Using rules 2 and 4*
300.00 mm has five significant figures.	*Using rules 1, 2, and 3*
5.06 s has three significant figures.	*Using rules 1 and 3*
304 s has three significant figures.	*Using rules 1 and 3*
0.0060 mm has two significant figures (6 and the last 0).	*Using rules 1, 2, and 4*
140 mm has two significant figures (just 1 and 4).	*Using rules 1 and 4*

PRACTICE PROBLEMS

1. State the number of significant figures in each measurement.
   a. 1405 m
   b. 2.50 km
   c. 0.0034 m
   d. 12.007 kg
   e. 5.8×10^{6} kg
   f. 3.03×10^{25} mL

There are two cases in which numbers are considered exact and, thus, have an infinite number of significant figures.

1. Counting numbers have an infinite number of significant figures.
2. Conversion factors have an infinite number of significant figures.

Examples:

The factor "2" in 2*mg* has an infinite number of significant figures.

The number 2 is a counting number. It is an exact integer.

The number "4" in 4 electrons has an infinite number of significant figures.

Because you cannot have a partial electron, the number 4, a counting number, is considered to have an infinite number of significant figures.

60 s/1 min has an infinite number of significant figures.

There are exactly 60 seconds in 1 minute, thus there is an infinite number of significant figures in the conversion factor.

Rounding

You can round a number to a specific place value (such as hundreds or tenths) or to a specific number of significant figures. To do this, determine the place to which you are rounding, and then use the following rules.

1. When the leftmost digit to be dropped is less than 5, that digit and any digits that follow are dropped. Then the last digit in the rounded number remains unchanged.
2. When the leftmost digit to be dropped is greater than 5, that digit and any digits that follow are dropped, and the last digit in the rounded number is increased by one.
3. When the leftmost digit to be dropped is 5 followed by a nonzero number, that digit and any digits that follow are dropped. The last digit in the rounded number increases by one.
4. If the digit to the right of the last significant digit is equal to 5 and 5 is followed by a zero or no other digits, look at the last significant digit. If it is odd, increase it by one; if it is even, do not round up.

Examples: Round the following numbers to the stated number of significant figures.

8.7645 rounded to 3 significant figures is 8.76.	*Using rule 1*
8.7676 rounded to 3 significant figures is 8.77.	*Using rule 2*
8.7519 rounded to 2 significant figures is 8.8.	*Using rule 3*
92.350 rounded to 3 significant figures is 92.4.	*Using rule 4*
92.25 rounded to 3 significant figures is 92.2.	*Using rule 4*

PRACTICE PROBLEMS

2. Round each number to the number of significant figures shown in parentheses.

- **a.** 1405 m (2)
- **b.** 2.50 km (2)
- **c.** 0.0034 m (1)
- **d.** 12.007 kg (3)

Operations with Significant Figures

If using a calculator, do all of the operations with as much precision as the calculator allows, and then round the result to the correct number of significant figures. The number of significant figures in the result depends on the measurements and on the operation.

Addition and subtraction Round the result to the least precise value among the measurements—the smallest number of digits to the right of the decimal points.

Example: Add 1.456 m, 4.1 m, and 20.3 m.

The least precise values are 4.1 m and 20.3 m because they have only one digit to the right of the decimal points.

1.456 m *Add the numbers.*
4.1 m
+ 20.3 m
25.856 m

25.9 m *Round the result to place value of the least precise value.*

Multiplication and division Look at the number of significant figures in each measurement. Perform the calculation. Round the result so that it has the same number of significant figures as the measurement with the least number of significant figures.

Example: Multiply 20.1 m by 3.6 m.

$$(20.1\text{ m})(3.6\text{ m}) = 72.36\text{ m}^2$$

The least precise value is 3.6 m with two significant figures. The product can only have as many digits as the least precise of the multiplied numbers.

72 m   Round the result to two significant figures.

PRACTICE PROBLEMS

3. Simplify the following expressions using the correct number of significant figures.

a. 5.012 km + 3.4 km + 2.33 km

b. 45 g − 8.3 g

c. 3.40 cm × 7.125 cm

d. 54 m ÷ 6.5 s

Combination When doing a calculation that requires a combination of addition/subtraction and multiplication/division, use the multiplication/division rule.

Examples:

$$x = 19\text{ m} + (25.0\text{ m/s})(2.50\text{ s}) + \frac{1}{2}(-10.0\text{ m/s}^2)(2.50\text{ s})^2$$

$$= 5.0\times10^1\text{ m}$$

19 m only has two significant figures, so the answer should only have two significant figures.

$$\text{slope} = \frac{70.0\text{ m} - 10.0\text{ m}}{29\text{ s} - 11\text{ s}}$$

$$= 3.3\text{ m/s}$$

29 s and 11 s only have two significant figures each, so the answer should only have two significant figures.

Multistep calculations Do not round to significant figures in the middle of a multistep calculation. Instead, round to a reasonable number of decimal places that will not cause you to lose significance in your answer. When you get to your final step where you are solving for the answer asked for in the question, you should then round to the correct number of significant figures.

Example:

$$F = \sqrt{(24\text{ N})^2 + (36\text{ N})^2}$$

$$= \sqrt{576\text{ N}^2 + 1296\text{ N}^2}$$

Do not round to 580 N^2 and 1300 N^2.

$$= \sqrt{1872\text{ N}^2}$$

Do not round to 1800 N^2.

$$= 43\text{ N}$$

Final answer, so it should be rounded to two significant figures.

If you had rounded in each step, you would have obtained an answer of 44 N. This might seem like a small discrepancy, but in more complex calculations, the effects can be large. For example, if you calculate 5 × 5 × 5 × 5 × 5 without rounding your answer will be 3125, which rounds to 3000. If you round at each step, however, the answer you find is 2500, which rounds to 2000.

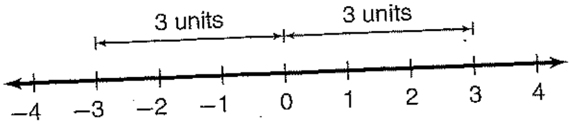
Absolute Value

The absolute value of a number (n) is its magnitude, regardless of its sign. The absolute value of n is written as $|n|$. Because magnitudes cannot be less than zero, absolute values always are greater than or equal to zero.

Examples:

$|3| = 3$

$|-3| = 3$



V. Scientific Notation

A number of the form $a\times10^n$ is written in scientific notation, where $1 \leq a \leq 10$, and n is an integer. The base (10) is raised to a power (n).

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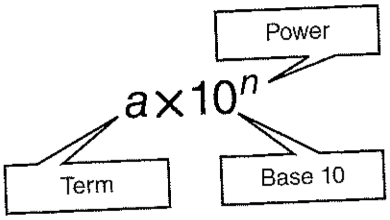

Physicists commonly use scientific notation to express measurements that are greater than 10 or less than 1. For example, the mass of a proton is written as 6.73×10^{-28} kg. The density of water is written as 1.000×10^3 kg/m^3. This shows, using significant digit rules, that this measurement is exactly 1000 to four significant figures. However, writing the density of water as 1000 kg/m^3 would imply that it has only one significant digit, which is incorrect. Scientific notation helps physicists keep accurate track of significant figures.

Large Numbers—Using Positive Exponents

Multiplying by a power of 10 is like moving the decimal point that same number of places to the right (if the power is positive) or to the left (if the power is negative). To express a large number in scientific notation, first determine the value for a, $1 \leq a \leq 10$. Count the number of decimal places from the decimal point in a to the decimal point in the number. Use that count as the power of 10. A calculator shows scientific notation with e for exponent, as in $2.4e + 11 = 2.4\times10^{11}$. Some calculators use an E to show the exponent, or there is often a place on the display where the calculator can show smaller-sized digits representing the exponent.

Example: Write 7,530,000 in scientific notation.

The value for a is 7.53. (The decimal point is to the right of the first nonzero digit.) So the form will be 7.53×10^n.

$7{,}\underset{\longleftarrow}{\underbrace{530{,}000}} = 7.53\times10^6$ *There are six decimal places, so the power is 6.*

To write the standard form of a number expressed in scientific notation, write the value of a, and place extra zeros to the right of the number. Use the power and move the decimal point in a that many places to the right.

Example: Write the following number in standard form.

$2.389\times10^5 = 2.38900\times10^5 = 2\underset{\longrightarrow}{\underbrace{38{,}900}}$

Small Numbers—Using Negative Exponents

To express a small number in scientific notation, first determine the value for a, $1 \leq a \leq 10$. Then count the number of decimal places from the decimal point in a to the decimal point in the number. Use that number as the power of 10. Multiplying by a number with a negative power is the same as dividing by that number with the corresponding positive power.

Example: Write 0.000000285 in scientific notation.

The value for a is 2.85. (The decimal point is to the right of the first nonzero digit.) So the form will be 2.85×10^{n}.

$0.\underbrace{0000002}_{\longrightarrow}85 = 2.85\times10^{-7}$ *There are seven decimal places, so the power is −7.*

To express a small number in standard form, write the value for a and place extra zeros to the left of a. Use the power and move the decimal point in a that many places to the left.

Example:

$1.6\times10^{-4} = 00001.6\times10^{-4} = 0.\underbrace{0001}_{\longleftarrow}6$

PRACTICE PROBLEMS

11. Express each number in scientific notation.

a. 456,000,000 **b.** 0.000020

12. Express each number in standard notation.

a. 3.03×10^{-7} **b.** 9.7×10^{10}

Operations with Scientific Notation

Calculating with numbers written in scientific notation uses the properties of exponents.

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Multiplication Multiply the terms, and add the powers of 10.

Example: Simplify.

$$\begin{aligned}(4.0\times10^{-8})(1.2\times10^{5}) &= (4.0 \times 1.2)(10^{-8}\times10^{5}) && \textit{Group terms and bases of 10.}\\ &= (4.8)(10^{-8+5}) && \textit{Multiply terms.}\\ &= (4.8)(10^{-3}) && \textit{Add powers of 10.}\\ &= 4.8\times10^{-3} && \textit{Recombine in scientific notation.}\end{aligned}$$

Division Divide the base numbers and subtract the exponents of 10.

Example: Simplify.

$$\begin{aligned}\frac{9.60\times10^{7}}{1.60\times10^{3}} &= \left(\frac{9.60}{1.60}\right)\times\left(\frac{10^{7}}{10^{3}}\right) && \textit{Group terms and bases of 10.}\\ &= 6.00\times10^{7-3} && \textit{Divide terms, and subtract powers of 10.}\\ &= 6.00\times10^{4}\end{aligned}$$

Addition and subtraction To add and subtract numbers in scientific notation, the powers of 10 must be the same. Thus, you may need to rewrite one of the numbers with a different power of 10. With equal powers of 10, use the distributive property.

Example: Simplify.

$$\begin{aligned}(3.2\times10^5) + (4.8\times10^5) &= (3.2 + 4.8)\times10^5 && \text{Group terms.}\\ &= 8.0\times10^5 && \text{Add terms.}\end{aligned}$$

Example: Simplify.

$$\begin{aligned}(3.2\times10^5) + (4.8\times10^4) &= (3.2\times10^5) + (0.48\times10^5) && \text{Rewrite } 4.8\times10^4 \text{ as } 0.48\times10^5.\\ &= (3.2 + 0.48)\times10^5 && \text{Group terms.}\\ &= 3.68\times10^5 && \text{Add terms.}\\ &= 3.7\times10^5 && \text{Round using the addition significant figures rule.}\end{aligned}$$

PRACTICE PROBLEMS

13. Evaluate each expression; express the result in scientific notation.

a. $(5.2\times10^{-4})(4.0\times10^8)$

b. $(2.4\times10^3) + (8.0\times10^4)$

VI. Equations

Order of Operations

Scientists and mathematicians have agreed on a set of steps or rules, called the order of operations, so that everyone interprets mathematical symbols in the same way. Follow these steps in order when you evaluate an expression or use a formula.

1. Simplify the expressions inside grouping symbols, such as parentheses ( ), brackets [ ], braces { }, and fraction bars.
2. Evaluate all powers and roots.
3. Do all multiplications and/or divisions from left to right.
4. Do all additions and/or subtractions from left to right.

A useful way to remember these is the phrase "*P*lease *e*xcuse *m*y *d*ear *A*unt *S*ally." The first letter of each word represents an operation: *p*arentheses, *e*xponents, *m*ultiplication, *d*ivision, *a*ddition, *s*ubtraction.

Example: Simplify the following expression.

$$\begin{aligned}4 + 3(4 - 1) - 2^3 &= 4 + 3(3) - 2^3 && \text{Order of operations step 1.}\\ &= 4 + 3(3) - 8 && \text{Order of operations step 2.}\\ &= 4 + 9 - 8 && \text{Order of operations step 3.}\\ &= 5 && \text{Order of operations step 4.}\end{aligned}$$

The previous example was shown step-by-step to demonstrate the order of operations. When solving a physics problem, do not round to the correct number of significant figures until after the final calculation.

In calculations involving an expression in a numerator and an expression in a denominator, the numerator and the denominator are separate groups and you should calculate them before dividing the numerator by the denominator. The multiplication/division rule for significant figures is used to determine the final number of significant figures.

Solving Equations

To solve an equation means to find the value of the variable that makes the equation a true statement. To solve equations, apply the distributive property and the properties of equality. Any properties of equalities that you apply on one side of an equation, you also must apply on the other side.

Get help with using **the distributive property**

Distributive property For any numbers a, b, and c,

$$a(b + c) = ab + ac \qquad a(b - c) = ab - ac$$

Example: Use the distributive property to expand the following expression.

$$3(x + 2) = 3x + (3)(2) = 3x + 6$$

Addition and subtraction properties of equality If two quantities are equal, and the same number is added to (or subtracted from) each, then the resulting quantities also are equal.

If $a = b$, then

$$a + c = b + c \quad \text{and} \quad a - c = b - c$$

Example: Solve $x - 3 = 7$ using the addition property.

$$\begin{aligned} x - 3 &= 7 \\ x - 3 + 3 &= 7 + 3 \\ x &= 10 \end{aligned}$$

Example: Solve $t + 2 = -5$ using the subtraction property.

$$\begin{aligned} t + 2 &= -5 \\ t + 2 - 2 &= -5 - 2 \\ t &= -7 \end{aligned}$$

Multiplication and division properties of equality If two equal quantities each are multiplied by (or divided by) the same number, then the resulting quantities also are equal.

If $a = b$, then

$ac = bc$ and

$$\frac{a}{c} = \frac{b}{c}, \text{ for } c \neq 0$$

Example: Solve $\frac{1}{4}a = 3$ using the multiplication property.

$$\begin{aligned} \frac{1}{4}a &= 3 \\ \left(\frac{a}{4}\right)(4) &= 3(4) \\ a &= 12 \end{aligned}$$

Example: Solve $6n = 18$ using the division property.

$$\begin{aligned} 6n &= 18 \\ \frac{6n}{6} &= \frac{18}{6} \\ n &= 3 \end{aligned}$$

Example: Solve $2t + 8 = 5t - 4$ for t.

$$\begin{aligned} 2t + 8 &= 5t - 4 \\ 8 + 4 &= 5t - 2t \\ 12 &= 3t \\ 4 &= t \end{aligned}$$

Isolating a Variable

Suppose an equation has more than one variable. To isolate a variable—that is, to solve the equation for a variable—write an equivalent equation so that one side contains only that variable with a coefficient of 1.

Isolate the variable P (pressure) in the ideal gas law equation.

Get help with **isolating a variable.**

$$PV = nRT$$

$$\frac{PV}{V} = \frac{nRT}{V} \qquad \text{Divide both sides by } V.$$

$$P\left(\frac{V}{V}\right) = \frac{nRT}{V} \qquad \text{Group } \frac{V}{V}.$$

$$P = \frac{nRT}{V} \qquad \text{Substitute } \frac{V}{V} = 1.$$

PRACTICE PROBLEMS

14. Solve for x.

a. $2 + 3x = 17$

b. $x - 4 = 2 - 3x$

c. $t - 1 = \frac{x + 4}{3}$

d. $a = \frac{b + x}{c}$

e. $\frac{2x + 3}{x} = 6$

f. $ax + bx + c = d$

Square Root Property

If a and n are real numbers, $n > 0$, and $a^2 = n$, then $a = \pm\sqrt{n}$.

Solve for v in Newton's second law for a satellite orbiting Earth.

$$\frac{mv^2}{r} = \frac{Gm_Em}{r^2}$$

$$\frac{rmv^2}{r} = \frac{rGm_Em}{r^2} \qquad \text{Multiply both sides by } r.$$

$$mv^2 = \frac{Gm_Em}{r} \qquad \text{Substitute } \frac{r}{r} = 1.$$

$$\frac{mv^2}{m} = \frac{Gm_Em}{rm} \qquad \text{Divide both sides by } m.$$

$$v^2 = \frac{Gm_E}{r} \qquad \text{Substitute } \frac{m}{m} = 1.$$

$$\sqrt{v^2} = \pm\sqrt{\frac{Gm_E}{r}} \qquad \text{Take the square root.}$$

$$v = \sqrt{\frac{Gm_E}{r}} \qquad \text{Use the positive value for speed.}$$

When using the square root property, it is important to consider what you are solving for. Because we solved for speed in the above example, it did not make sense to use the negative value of the square root. Also, you need to consider if the negative or positive value gives you a realistic solution. For example, when using the square root property to solve for time, a negative value might give you a time before the situation even started.

Quadratic Equations

A quadratic equation has the form $ax^2 + bx + c = 0$, where $a \neq 0$. A quadratic equation has one variable with a power (exponent) of 2. It also might include that same variable to the first power. You can estimate the solutions by graphing on a graphing calculator. If $b = 0$, then there is no x-term in the quadratic equation. In this case, you can solve the equation by isolating the squared variable and finding the square root of each side of the equation using the square root property.

Quadratic Formula

You can find the solutions of any quadratic equation by using the quadratic formula. The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

As with the square root property, it is important to consider whether the solutions to the quadratic formula give you a realistic answer to the problem you are solving. Usually, you can throw out one of the solutions because it is unrealistic. Projectile motion often requires the use of the quadratic formula when solving equations, so keep the realism of the solution in mind when solving.

Get help with using **the quadratic formula**

PRACTICE PROBLEMS

15. Solve for x.

- **a.** $4x^2 - 19 = 17$
- **b.** $12 - 3x^2 = -9$
- **c.** $x^2 - 2x - 24 = 0$
- **d.** $24x^2 - 14x - 6 = 0$

Dimensional Calculations

When doing calculations, you must include the units of each measurement that is written in the calculation. All operations that are performed on the number also are performed on its units.

The acceleration due to gravity (a) is given by the equation $a = \frac{2\Delta x}{\Delta t^2}$. A free-falling object near the Moon drops 20.5 m in 5.00 s. Find the acceleration (a). Acceleration is measured in meters per second squared.

$$a = \frac{2\Delta x}{\Delta t^2}$$

$$a = \frac{2(20.5\ \text{m})}{(5.00\ \text{s})^2}$$

The number 2 is an exact number, so it does not affect the determination of significant figures.

$$a = \frac{1.64\ \text{m}}{\text{s}^2} \text{ or } 1.64\ \text{m/s}^2$$

Calculate and round to three significant figures.

Unit conversion Use a conversion factor to convert from one measurement unit to another of the same type, such as from minutes to seconds. This is equivalent to multiplying by one.

Find Δx when $v_i = 67$ m/s and $\Delta t = 5.0$ min. Use the equation $\Delta x = v_i\Delta t$.

$$\frac{60 \text{ seconds}}{1 \text{ minute}} = 1$$

$$\Delta x = v_i\Delta t$$

$$\Delta x = \frac{67 \text{ m}}{\text{s}}\left(\frac{5.0 \text{ min}}{1}\right)\left(\frac{60 \text{ s}}{1 \text{ min}}\right)$$

Multiply by the conversion factor $\frac{60 \text{ s}}{1 \text{ min}} = 1$

$$\Delta x = 20{,}100 \text{ m} = 2.0\times10^4 \text{ m}$$

Calculate and round to two significant figures. The numbers 60 s and 1 min are exact numbers, so they do not affect the determination of significant figures.

PRACTICE PROBLEMS

16. Simplify $\Delta t = \frac{4.0\times10^2 \text{ m}}{16 \text{ m/s}}$.

17. Find the velocity of a dropped brick after 5.0 s using $v = a\Delta t$ and $a = -9.8$ m/s^2.

18. Calculate the product: $\left(\frac{32 \text{ cm}}{1 \text{ s}}\right)\left(\frac{60 \text{ s}}{1 \text{ min}}\right)\left(\frac{60 \text{ min}}{1 \text{ h}}\right)\left(\frac{1 \text{ m}}{100 \text{ cm}}\right)$.

19. An Olympian ran 100.0 m in 9.87 s. What was the speed in kilometers per hour?

Dimensional Analysis

Dimensional analysis is a method of doing algebra with the units. It often is used to check the validity of the units of a final result and the equation being used, without completely redoing the calculation.

Physics Example Verify that the final answer of $x_f = x_i + v_it + \frac{1}{2}at^2$ will have the units meters (m).

x_i is measured in meters (m).

t is measured in seconds (s).

v_i is measured in meters per second (m/s).

a is measured in meters per second squared (m/s^2).

Get help with **dimensional analysis.**

$$x_f = \text{m} + \left(\frac{\text{m}}{\text{s}}\right)(\text{s}) + \frac{1}{2}\left(\frac{\text{m}}{\text{s}^2}\right)(\text{s})^2$$

Substitute the units for each variable.

$$= \text{m} + \text{m}\left(\frac{\text{s}}{\text{s}}\right) + \frac{1}{2}(\text{m})\left(\frac{\text{s}^2}{\text{s}^2}\right)$$

Simplify the fractions using the distributive property.

$$= \text{m} + (\text{m})(1) + \frac{1}{2}(\text{m})(1)$$

Substitute s/s = 1, s²/s² = 1.

$$= \text{m} + \text{m} + \frac{1}{2}\text{m}$$

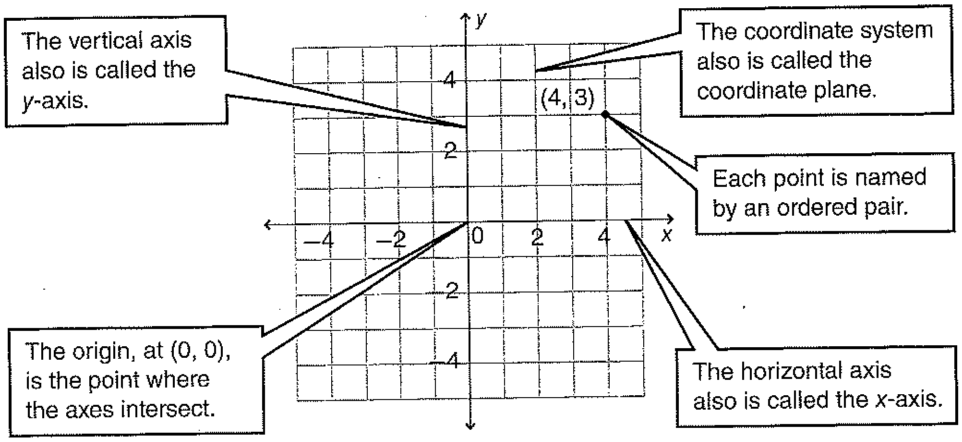
Everything simplifies to m; thus x_f is in m.

The factor of $\frac{1}{2}$ in the above does not apply to the units. It applies only to any number values that would be inserted for the variables in the equation. It is easiest to remove number factors such as the $\frac{1}{2}$ when first setting up the dimensional analysis.

VII. Graphs of Relations

The Coordinate Plane

You can locate points on a plane in reference to two perpendicular number lines, called axes. The horizontal number line is called the *x*-axis and represents the independent variable. The vertical number line is called the *y*-axis and represents the dependent variable. A point is represented by two coordinates (*x*, *y*), which also is called an ordered pair. The value of the independent variable (*x*) always is listed first in the ordered pair. The ordered pair (0, 0) represents the origin.



The vertical axis also is called the *y*-axis.

The coordinate system also is called the coordinate plane.

Each point is named by an ordered pair.

The origin, at (0, 0), is the point where the axes intersect.

The horizontal axis also is called the *x*-axis.

Get help

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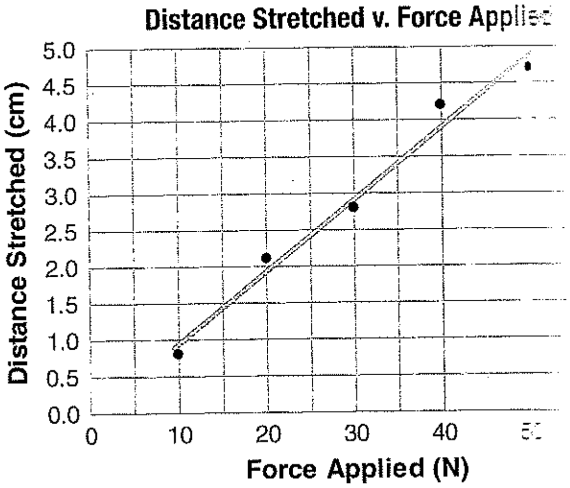
Graphing Data to Determine a Relationship

Use the following steps to graph data.

1. Identify the independent and the dependent variables.
2. Draw two perpendicular axes. Label each axis using the variable names.
3. Determine the range of data for each variable. Use the ranges to decide on a convenient scale for each axis. Mark and number the scales.
4. Plot each data point.
5. If the points seem to lie approximately in a line, draw a best-fit line. The line should be as close to as many points as possible. If the points do not lie in a line, draw a smooth curve through as many points as possible. If there does not appear to be a trend, do not draw any line or curve.
6. Write a title that clearly describes what the graph represents.

Get help

Applied Force (N)	Distance Stretched (cm)
10	0.8
20	2.1
30	2.8
40	4.2
50	4.7



Interpolating and Extrapolating

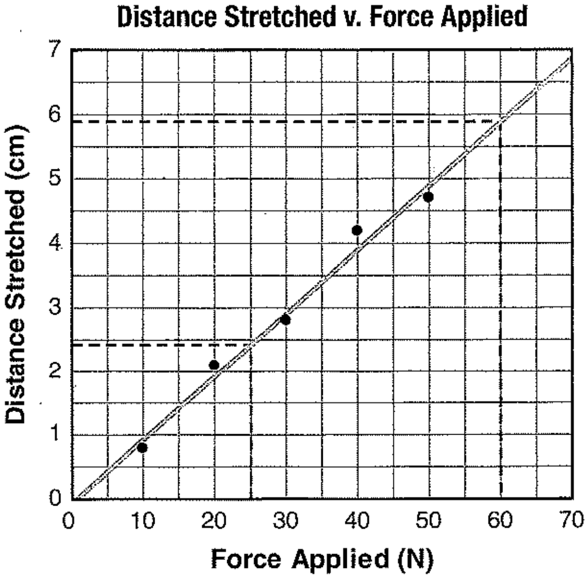
Interpolation is a process used to estimate a value for a relation that lies between two known values. Extrapolation is a process used to estimate a value for a relation that lies beyond the known values. The equation of the best-fit line helps you interpolate and extrapolate.

Example: Using the data and the graph, estimate how far the spring will stretch if a force of 25 N is applied. Use interpolation.

- Draw a best-fit line.
- Draw a line segment from 25 N on the *x*-axis to the best-fit line.
- Draw a line segment from that intersection point to the *y*-axis.
- Read the scale on the *y*-axis. A force of 25 N will stretch the spring 2.4 cm.

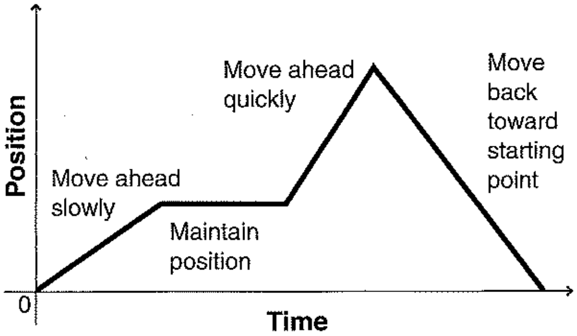
Example: Use extrapolation to estimate how far the spring will stretch if a 60-N force is applied.

- Draw a line segment from 60 on the *x*-axis to the best-fit line. Extend the best-fit line if necessary.
- Read the corresponding value on the *y*-axis. Extend the axis scale if necessary.
- A force of 60 N will stretch the spring 5.8 cm.

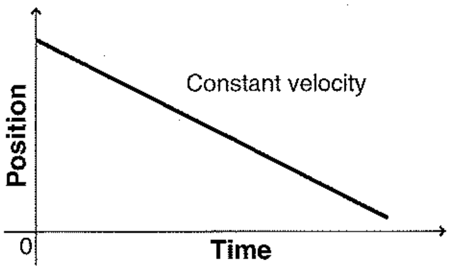


Interpreting Line Graphs

A line graph shows the linear relationship between two variables. Two types of line graphs that describe motion are used frequently in physics.



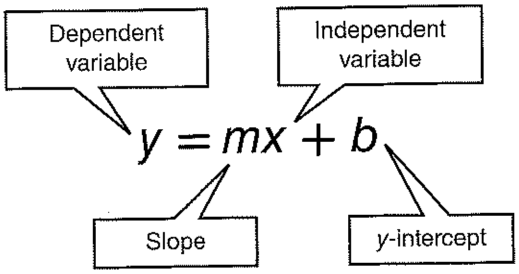
This line graph shows a changing relationship between the two graphed variables.



The line graph above shows a constant relationship between the two graphed variables.

Linear Equations

A linear equation can be written as a relation (or a function), $y = mx + b$, where m and b are real numbers, m represents the slope of the line, and b represents the intercept, the point at which the line crosses the y-axis.



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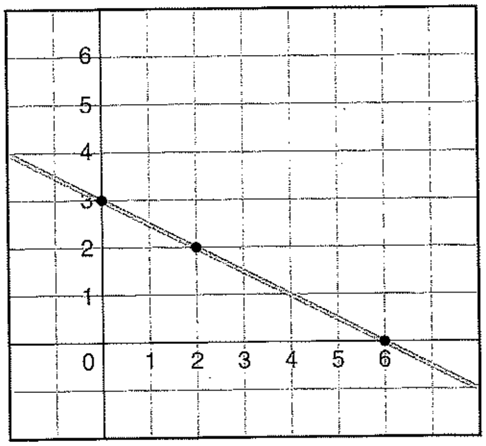
The graph of a linear equation is a line. The line represents all of the solutions of the linear equation. To graph a linear equation, choose three values for the independent variable. (Only two points are needed, but the third point serves as a check.) Calculate the corresponding values for the dependent variable. Plot each ordered pair (x, y) as a point. Draw a line through the points.

Example: Graph $y = -\frac{1}{2}x + 3$

Calculate three ordered pairs to obtain points to plot.

Ordered Pairs	
x	y
0	3
2	2
6	0

Ordered Pairs Plot



Slope

The slope of a line is the ratio of the change in y-coordinates to the change in x-coordinates. It is the ratio of the vertical change (rise) to the horizontal change (run). This number tells you how steep the line is. It can be a positive number or a negative number.

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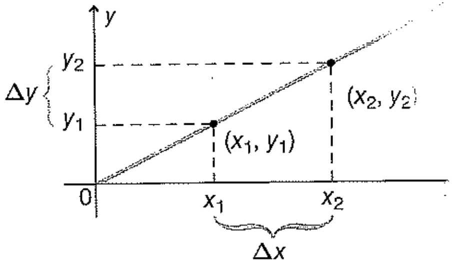
To find the slope of a line, select two points, (x_1, y_1) and (x_2, y_2). Calculate the run, which is the difference (change) between the two x-coordinates, $x_2 - x_1 = \Delta x$. Calculate the rise, which is the difference (change) between the two y-coordinates, $y_2 - y_1 = \Delta y$. Form the ratio.



$$\text{Slope } m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$
$$\text{where } x_1 \neq x_2$$

Direct Variation

If there is some nonzero constant (m) such that $y = mx$, then y varies directly with x. That means as the independent variable x doubles, the dependent variable y doubles. The variables x and y also are said to be proportional. This is a linear equation of the form $y = mx + b$ in which the value of b is zero. The graph passes through the origin, (0, 0).

In the force equation for an ideal spring, $F = -kx$, where F is the force on the spring, $-k$ is the spring constant, and x is the spring's displacement, the force on the spring varies directly with (is proportional to) the spring's displacement. That is, the force on the spring increases as the spring's displacement increases.

Inverse Variation

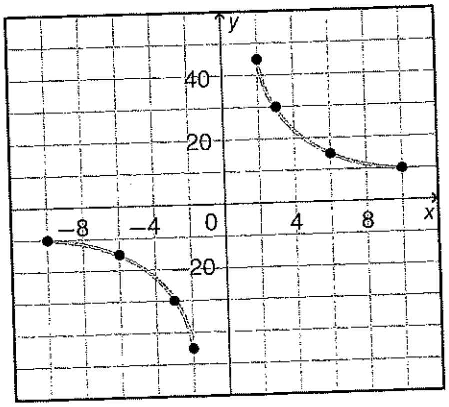
If there is some nonzero constant (m) such that $y = \frac{m}{x}$, then y varies inversely with x. That means as the independent variable x increases, the dependent variable y decreases. The variables x and y also are said to be inversely proportional. This is not a linear equation because it contains the product of two variables. The graph of an inverse relationship is a hyperbola. This relationship can be written as

$$y = \frac{m}{x}$$
$$y = m\frac{1}{x}$$
$$xy = m$$

Example: Graph the equation $y = \frac{90}{x}$.

Ordered Pairs	
x	**y**
−10	−9
−6	−15
−3	−30
−2	−45
2	45
3	30
6	15
10	9

Inverse Variation



In the equation for the speed of a wave ($\lambda = \frac{v}{f}$) where λ is wavelength, f is frequency, and v is wave speed, wavelength varies inversely with (is inversely proportional to) frequency. That is, as the frequency of a wave increases, the wavelength decreases. v is constant.

Quadratic Graphs

A quadratic relationship is a relationship of the form

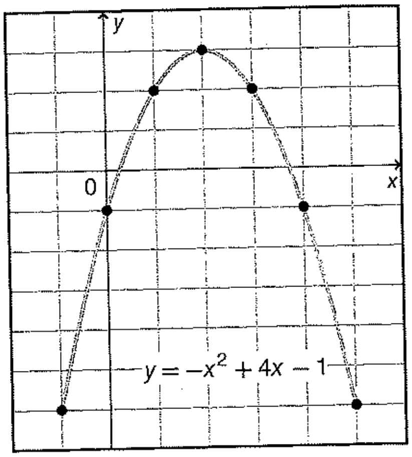
$$y = ax^2 + bx + c, \text{ where } a \neq 0.$$

A quadratic relationship includes the square of the independent variable (x). The graph of a quadratic relationship is a parabola. Whether the parabola opens upward or downward depends on whether the value of the coefficient of the squared term (a) is positive or negative.

Example: Graph the equation $y = -x^2 + 4x - 1$.

Ordered Pairs	
x	y
−1	−6
0	−1
1	2
2	3
3	2
4	−1
5	−6

Quadratic Equation Graph

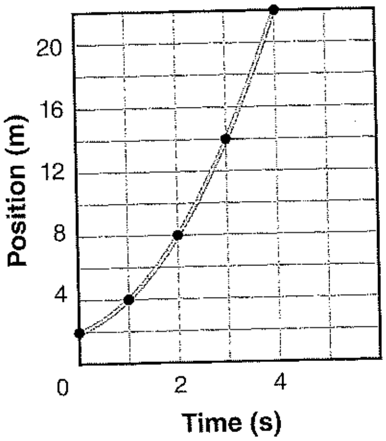


A position-time graph in the shape of a quadratic relation means that the object is moving at a constant acceleration.

$$x_f = 2\text{ m} + (1\text{ m/s})t + \frac{1}{2}(2\text{ m/s}^2)t^2$$

Ordered Pairs	
Time (s)	Position (m)
0	2
1	4
2	8
3	14
4	22

Position v. Time



VIII. Geometry and Trigonometry

Perimeter and Area

	Perimeter, Circumference Linear units	**Area** Squared units
Square side a	$P = 4a$	$A = a^2$
Rectangle length l width w	$P = 2l + 2w$	$A = lw$
Triangle base b height h	$P =$ side 1 + side 2 + side 3	$A = \frac{1}{2}bh$
Circle radius r	$C = 2\pi r$	$A = \pi r^2$

Surface Area and Volume

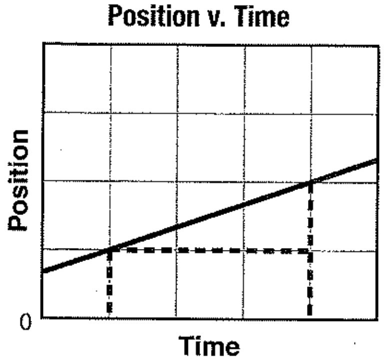
	Surface Area Squared units	**Volume** Cubic units
Cube side a	$SA = 6a^2$	$V = a^3$
Cylinder radius r height h	$SA = 2\pi rh + 2\pi r^2$	$V = \pi r^2 h$
Sphere radius r	$SA = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$

Look for geometric shapes in your physics problems. They could be in the form of objects or spaces. For example, vectors can sometimes form two-dimensional shapes.

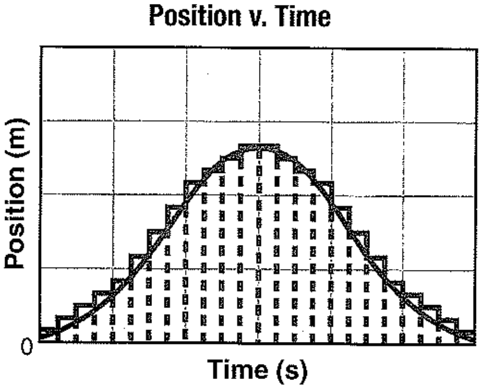
Area Under a Graph

To calculate the approximate area under a graph, cut the area into smaller pieces and find the area of each piece using the formulas shown above. To approximate the area under a line, cut the area into a rectangle and a triangle, as shown below on the left.

To approximate the area under a curve, draw several rectangles from the x-axis to the curve, as shown below on the right. Using more rectangles with a smaller base will provide a closer approximation of the area.



Total area = Area of the rectangle + Area of the triangle



Total area = Area 1 + Area 2 + Area 3 + …