CHAPTER 9

# Momentum and Its Conservation

## Air Force

Some cars already have air bags. In a head-on crash of two air bag-equipped cars, both drivers walked away uninjured. How does an air bag help to reduce the injury to a person in a car crash?

### Chapter Outline



In the crash of a car moving at high speed, the passengers are brought to a stop very quickly. You have learned that force is needed to produce the acceleration of the passengers. We will now take a slightly different point of view. Rather than studying forces acting on an object, and the acceleration that results, we will look at a clearly defined collection of objects before and after an interaction takes place. We will look for properties that remain constant. Such properties are said to be conserved.

### Concept Check

The following terms or concepts from earlier chapters are important for a good understanding of this chapter. If you are not familiar with them, you should review them before studying this chapter.
- velocity, Chapter 3
- mass, force, Newton's second and third laws, Chapter 5
- vectors, Chapter 6

# Objectives

- define momentum and impulse and use the momentum-impulse theorem to calculate changes in momentum; understand the relation between average force and time interval for a fixed impulse.
- recognize the connection between the third law and conservation of momentum; use the definition of a closed, isolated system.

# 9.1 IMPULSE AND CHANGE IN MOMENTUM

The word momentum is often used in everyday speech. A winning sports team is said to have the "big mo," momentum—as is a politician who rates high in opinion polls. In physics, however, momentum has a very special definition. Newton actually wrote his three laws of motion in terms of momentum, which he called the *quantity of motion*.

## Momentum and Impulse

Suppose a heavy truck and a compact car move down the road at the same velocity. If the two stop in the same time interval, it takes more force to stop the more massive truck. Now consider two cars of equal mass. If one car is moving faster than the other, a larger force is needed to stop the faster car in the same time interval. Obviously both the velocity and the mass of a moving object help determine what is needed to change its motion. The product of the mass and velocity of a body is called **momentum.** Momentum is a vector quantity that has the same direction as the velocity of the object.

Momentum is the product of the mass and the velocity of a body. It is a vector quantity.

Momentum is represented by $p$. The equation for momentum is

$$\vec{p} = m\vec{v}.$$

The unit for momentum is kilogram·metre/second (kg·m/s).

According to Newton's first law, if no net force acts on a body, its velocity is constant. If we consider only a single isolated object, then its mass cannot change. If its velocity is also constant, then so is its momentum, the product of its mass and velocity. In other words, if a single body has no net force acting on it, its momentum is constant. Its momentum is conserved.

Newton's second law describes how the velocity of a body is changed by a force acting on it, Figure 9–1. Let's rewrite Newton's second law using the definition of acceleration as the change in velocity divided by the time interval,

$$\vec{F} = m\vec{a} = \frac{m\Delta\vec{v}}{\Delta t}.$$

**POCKET LAB**

**CART MOMENTUM**

Attach a spring scale to a laboratory cart. First, pull the cart for 1.0 s exerting 1.0 N of force. Second, pull the cart for 2.0 s exerting about 0.50 N of force. Predict which trial will give the cart more acceleration. Explain. Predict which trial will give the cart more velocity. Explain. Try it. What happened? Which factor, $F$ or $\Delta t$, seems to be more important in changing the velocity of the cart?

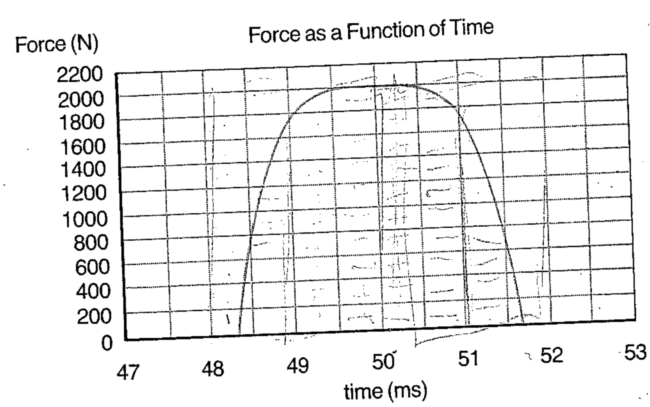



**FIGURE 9–1. The graph shows the force exerted by a moving ball colliding with a ball at rest.**

Multiplying both sides of the equation by $\Delta t$, we have the equation

$$\vec{F}\Delta t = m\Delta\vec{v}.$$

The left side of this equation, the product of the net force and the time interval over which it acts, is called the **impulse.** Impulse is a vector in the direction of the force. Impulse is measured in units of newton·second (N·s).

If the mass of an object is constant, then a change in its velocity results in a change in its momentum. Under these conditions,

$$\Delta\vec{p} = m\Delta\vec{v}.$$

Thus, we see that the impulse given to an object is equal to the change in its momentum,

$$\vec{F}\Delta t = \Delta\vec{p}.$$

This equation is called the **impulse-momentum theorem.** The equality of impulse and change in momentum is another way of writing Newton's second law. Often the force is not constant during the time it is exerted. In that case, the average force is used in the impulse-momentum theorem.

Impulse is the product of the average force and the time interval during which the force is exerted. A large change in momentum occurs only when there is a large impulse. A large impulse, however, can result from either a large force acting over a short time, or a smaller force acting over a longer time. What happens to the driver when a crash suddenly stops a car? An impulse is needed to bring the driver's momentum to zero. The steering wheel can exert a large force during a short length of time. An air bag reduces the force exerted on the driver by greatly increasing the length of time the force is exerted. The product of the average force and time interval of the crash would be the same.

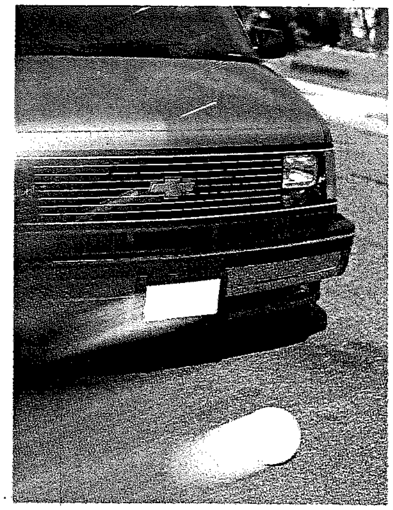


**FIGURE 9–2. With an equal amount of force, it will take longer to stop a truck than a compact car.**

Air Force

# Example Problem

## Calculating Momentum

A baseball of mass 0.14 kg is moving at +35 m/s. **a.** Find the momentum of the baseball. **b.** Find the velocity at which a bowling ball, mass 7.26 kg, would have the same momentum as the baseball.

**a. Given:** mass, $m = 0.14$ kg
velocity, $\vec{v} = +35$ m/s

**Unknown:** momentum, $\vec{p}$
**Basic equation:** $\vec{p} = m\vec{v}$

**Solution:** $\vec{p} = m\vec{v} = (0.14\text{ kg})(+35\text{ m/s}) = +4.9\text{ kg}\cdot\text{m/s}$

Note that the momentum is in the same direction (+) as the velocity.

**b. Given:** momentum,
$\vec{p} = +4.9$ kg·m/s
mass, $m = 7.26$ kg

**Unknown:** velocity, $\vec{v}$
**Basic equation:** $\vec{p} = m\vec{v}$

**Solution:** $\vec{p} = m\vec{v} \quad \vec{v} = \dfrac{\vec{p}}{m} = \dfrac{+4.9\text{ kg}\cdot\text{m/s}}{7.26\text{ kg}} = +0.67\text{ m/s}$

Note that the velocity is in the same (+) direction as the momentum.

Impulse is the product of a force and the time interval over which it acts.

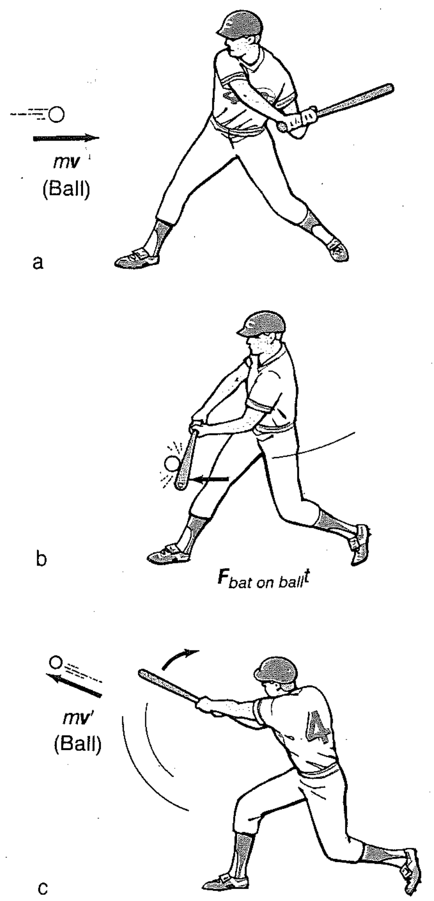



**FIGURE 9–3. The ball moves in the direction of the batter with momentum *mv*. The bat provides impulse $F_{\text{bat on ball}}t$ to the ball. The ball moves off with a change in momentum *mv'* equal to the impulse from the bat.**

## Example Problem

### Impulse and Momentum Change

A 0.144-kg baseball is pitched horizontally at +38 m/s. After it is hit by a bat, it moves horizontally at −38 m/s.

**a.** What impulse did the bat deliver to the ball?

**b.** If the bat and ball were in contact 0.80 ms, what was the average force the bat exerted on the ball?

**c.** Find the average acceleration of the ball during its contact with the bat.

Indicate the direction of the impulse, force, and acceleration.

**a. Given:** $m = 0.144$ kg
initial velocity,
$\vec{v}_i = +38$ m/s
final velocity,
$\vec{v}_f = -38$ m/s

**Unknown:** impulse, $\vec{F}\Delta t$
**Basic equation:** $\vec{F}\Delta t = \Delta\vec{p}$

**Solution:**
$$\begin{aligned}\vec{F}\Delta t &= \Delta\vec{p} = m\vec{v}_f - m\vec{v}_i = m(\vec{v}_f - \vec{v}_i)\\ &= (0.144\text{ kg})(-38\text{ m/s} - (+38\text{ m/s}))\\ &= (0.144\text{ kg})(-76\text{ m/s})\\ &= -11\text{ kg}\cdot\text{m/s (in the direction of the batted ball)}\end{aligned}$$

**b. Given:** momentum change,
$\Delta\vec{p} = 11$ kg·m/s
time interval,
$\Delta t = 0.80\text{ ms} = 8.0 \times 10^{-4}$ s

**Unknown:** average force, $\vec{F}$
**Basic equation:** $\vec{F}\Delta t = \Delta\vec{p}$

**Solution:** $\vec{F}\Delta t = \Delta\vec{p}$, so
$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t} = \frac{-11\text{ kg}\cdot\text{m/s}}{0.80\text{ ms}} = \frac{-11\text{ kg}\cdot\text{m/s}}{8.0 \times 10^{-4}\text{ s}}$$
$$= -1.4 \times 10^2\text{ N (in the direction of the batted ball)}$$

**c. Given:** average force
$\vec{F} = -1.4 \times 10^4$ N
mass,
$m = 0.144$ kg

**Unknown:** average acceleration, $\vec{a}$
**Basic equation:** $\vec{F} = m\vec{a}$

**Solution:** $\vec{F} = m\vec{a}$, so
$$\begin{aligned}\vec{a} &= \frac{\vec{F}}{m}\\ &= \frac{-1.4 \times 10^4\text{ N}}{0.144\text{ kg}}\\ &= -9.7 \times 10^4\text{ m/s}^2,\end{aligned}$$
about 10 000 gs in the direction of the batted ball.

## Practice Problems

1. A compact car, mass 725 kg, is moving at +100 km/h.
   **a.** Find its momentum.
   **b.** At what velocity is the momentum of a larger car, mass 2175 kg, equal to that of the smaller car?

2. A snowmobile has a mass of $2.50 \times 10^2$ kg. A constant force is exerted on it for 60.0 s. The snowmobile's initial velocity is 6.00 m/s and its final velocity 28.0 m/s.
   a. What is its change in momentum?
   b. What is the magnitude of the force exerted on it?
3. The brakes exert a $6.40 \times 10^2$ N force on a car weighing 15 680 N and moving at 20.0 m/s. The car finally stops.
   a. What is the car's mass?
   b. What is its initial momentum?
   c. What is the change in the car's momentum?
   d. How long does the braking force act on the car to bring it to a halt?
4. Figure 9–1 shows, as a function of time, the force exerted by a ball that collided with a box at rest. The impulse, $\vec{F}\Delta t$, is the area under the curve.
   a. Find the impulse given to the box by the ball.
   b. If the box has a mass of 2.4 kg, what velocity did it have after the collision?

## FINE ARTS CONNECTION

You often see mobiles at a museum or mall. Mobiles are made of different shaped objects that depend on the equilibrium of forces and torques to balance and move with air currents. The shape of mobiles and the shadows they make are part of their artistic value. However, the aesthetic value of a mobile depends on its movement as the swinging parts cut shapes in the air. The sculptor Alexander Calder was the first artist to make a mobile in which movement was the basic purpose. The next time you see a mobile, look for the relationship of space shapes to each other and to the mobile.

## Angular Momentum

As we have seen, the speed of an object moving in a circle changes only if a torque is applied to it. The quantity of angular motion that is similar to linear momentum is called **angular momentum.** Thus, if there is no torque on an object, its angular momentum is constant.

Momentum is the product of the object's mass and velocity. Angular momentum is the product of its mass, velocity, and distance from the center of rotation, and the component of velocity perpendicular to that distance. For example, the angular momentum of planets around the sun is constant. Therefore, when a planet's distance from the sun is larger, its velocity must be smaller. This is another statement of Kepler's second law.

Angular momentum is the product of an object's mass, velocity, and distance from centre of rotation.

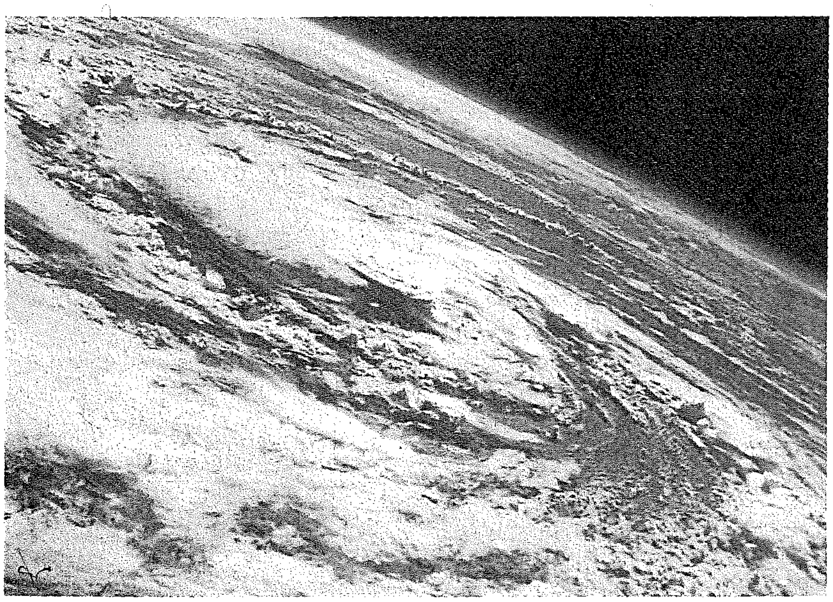


**FIGURE 9–4. This hurricane was photographed from space. The huge rotating mass of air possesses a large angular momentum.**

### Vector Conventions

- *Velocity vectors are red.*
- *Displacement vectors are green.*
- *Acceleration vectors are purple.*
- *Force vectors are blue.*
- ***Momentum vectors are orange.***
- *Vector resultants are dashed lines.*

## Objectives

- **state the law of conservation of momentum and use it, especially in collision problems.**
- **distinguish between external and internal forces and use this distinction.**
- **explain the extension of the law of conservation of momentum to two dimensions; solve collision problems using vectors.**

## CONCEPT REVIEW

**1.1** Is the momentum of a car traveling south different from that of the same car moving north at the same speed? Explain.

**1.2** If you jump off a table, as your feet hit the floor you let your legs bend at the knees. Explain why.

**1.3** Which has more momentum, a supertanker tied at the dock or a falling raindrop?

**1.4** **Critical Thinking:** An archer shoots arrows at a target. Some arrows stick in the target while others bounce off. Assuming the mass and velocity are the same, which arrows give a bigger impulse to the target? **Hint:** Consider the change in momentum of the arrow.

# 9.2 THE CONSERVATION OF MOMENTUM

Suppose you place several sugar cubes in a box and close it. We can call the box and the sugar in it a **system,** a defined collection of objects. Shake the box hard for several minutes. When you open it, you find that the shapes of the cubes have changed. In addition, there are sugar grains in the box that were not there before. It would be almost impossible to apply Newton's laws to each of the forces that were acting while the box was being shaken. Instead, we can look for a property of the system that has remained constant. A balance would show that the mass of the sugar and the box remains the same. The mass is conserved. Over the past century, physicists have found that studying conserved properties has produced great success in solving problems and understanding the principles of the physical world.

## Newton's Third Law and Momentum

The impulse-momentum theorem is another way of stating Newton's third law.

The example of a batted ball involved a type of collision. The baseball collided with the bat. In the collision, the momentum of the baseball changed as a result of the impulse given it by the bat. But, what happened to the bat? The bat exerted a force on the ball. By Newton's third law, the ball must have exerted a force on the bat of equal magnitude, but in the opposite direction. Thus, the bat also received an impulse. The direction of force on the bat is opposite the force on the ball, so the impulse given to the bat must also be in the opposite direction. By the impulse-momentum theorem, we know that the momentum of the bat must have changed. Its forward momentum was reduced as a result of its collision with the ball.

A system is isolated if no net external force acts on it.

The momentum change of the bat may not be obvious because the bat is massive and is held by a batter who stands with his feet firmly planted on the ground, usually wearing spiked shoes for a firmer grip. In the ball and bat collision, as with almost all natural processes, many forces are exerted. We need to look at a much simpler system to really understand the impulses and changes in momentum.

To study momentum changes in collisions, we must use a **closed, isolated system.** Remember, a system can be any specified collection of

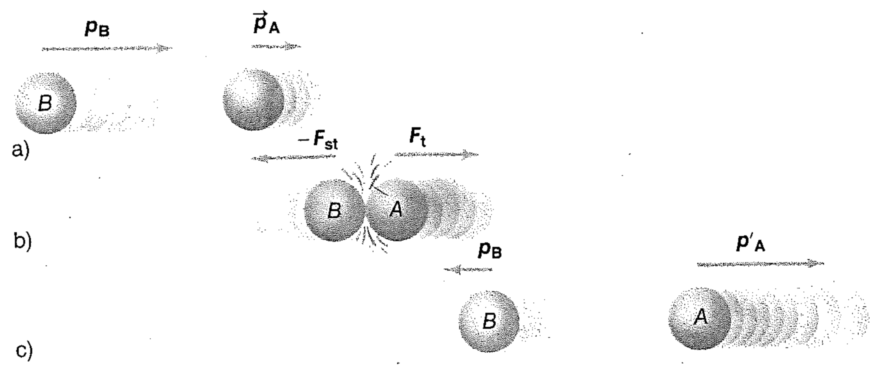



**FIGURE 9–5. Ball A is moving with momentum $\vec{p}_A$ while ball B is moving with momentum $\vec{p}_B$ (a). When they collide, the impulses provided to the balls are equal in magnitude, but opposite in direction (b). After the collision, ball A moves with new momentum $\vec{p}'_A$, while ball B moves with new momentum $\vec{p}'_B$ (c).**

objects. A system is closed if objects neither enter nor leave it. It is isolated if no net external force is exerted on it. Two balls on a billiard table are an isolated system as long as friction is small enough to be ignored, and as long as neither ball hits the bumper at the edge of the table.

In Figure 9–5, ball A is moving with momentum $\vec{p}_A$ and ball B with momentum $\vec{p}_B$. They collide, and ball B exerts force $+\vec{F}$ on ball A. The balls are in contact for a time $\Delta t$, so the impulse given ball A is $+\vec{F}\Delta t$. The momentum of ball A is changed by an amount equal to the impulse, $\Delta\vec{p} = +\vec{F}\Delta t$. The new momentum of ball A is

$$\vec{p}'_A = \vec{p}_A + \Delta\vec{p}.$$

During the collision, ball A also exerts a force on ball B. According to Newton's third law, the force ball A exerts on ball B is equal in magnitude but opposite in direction to the force B exerts on A. The time interval of the collision is the same, so the impulse given ball B is $-\vec{F}\Delta t$. The momentum of B is changed by an amount $-\Delta\vec{p}$ that is equal in size but opposite in direction to the momentum change of ball A; $-\Delta\vec{p} = -\vec{F}\Delta t$. Thus the new momentum of ball B is

$$\vec{p}'_B = \vec{p}_B + (-\Delta\vec{p}).$$

The momentum of ball B decreases while the momentum of ball A increases. The momentum lost by B equals the momentum gained by A. For the whole system consisting of the two balls, *the net change in momentum is zero*. That is, the final momentum of the system equals the initial momentum of the system; $\vec{p}_A + \vec{p}_B = \vec{p}'_A + \vec{p}'_B$. The total momentum before the collision is the same as the total momentum after the collision. That is, the momentum of the system is not changed; it is conserved. In summary:

**Table 9–1**

| Object | Ball A | Ball B | System |
|---|---|---|---|
| Initial momentum | $\vec{p}_A$ | $\vec{p}_B$ | $\vec{p}_A + \vec{p}_B$ |
| Impulse | $+\vec{F}\Delta t$ | $-\vec{F}\Delta t$ | 0 |
| Momentum change | $+\Delta\vec{p}$ | $-\Delta\vec{p}$ | 0 |
| Final momentum | $\vec{p}'_A$ | $\vec{p}'_B$ | $\vec{p}'_A + \vec{p}'_B$ |

## POCKET LAB

### SKATEBOARD FUN

Have two students sit facing each other on skateboards or laboratory carts, approximately 3 to 5 m apart. Place a rope in their hands. Predict what will happen when one student pulls on the rope while the other just holds his end. Explain your prediction. Which person is exerting more force on the rope? Compare the amount of time that the force is acting on each person. Which person will have a greater change in momentum? Explain. Do it. Describe what really happened. Can you devise a method to pull only one student to the other?

# PHYSICS LAB: The Explosion

## Purpose

To investigate and compare the forces and change in momentum acting on different masses during an explosion.

## Materials

- 2 laboratory carts (one with a spring mechanism)
- 2 C-clamps
- 2 blocks of wood
- 20-N spring balance
- 0.50-kg mass
- stopwatch
- masking tape

## Procedure

1. Securely tape the 0.50-kg mass to cart 2 and then use the balance to determine the mass of each cart.
2. Arrange the equipment as shown in the diagram.
3. Predict the starting position so that the carts will hit the blocks at the same instant when the spring mechanism is released.
4. Place pieces of tape on the table at the front of the carts. (See sketch.)
5. Depress the mechanism to release the spring mechanism and explode the carts.
6. Notice which cart hits the block first.
7. Adjust the starting position until the carts hit at the same instant. (Remember to move the tapes.)

## Observations and Data

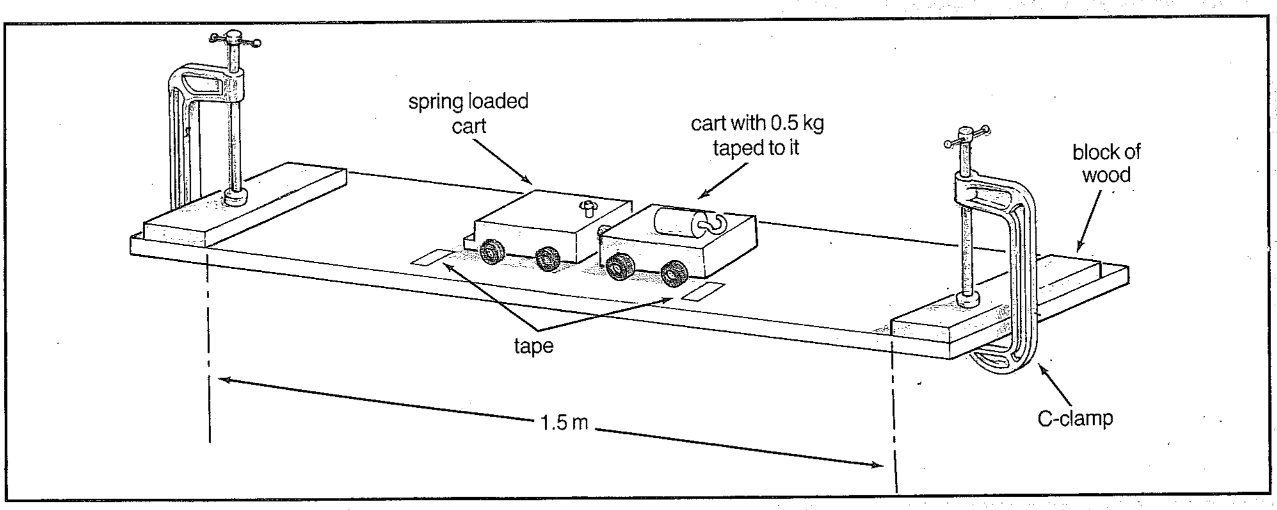
1. Which cart moved farther?
2. Which cart moved faster? Explain.

## Analysis

1. Estimate the velocity of each cart. Which was greater?
2. Compare the change in momentum of each cart.
3. Suppose that the spring pushed on cart 1 for 0.05 s. How long did cart 2 push on the spring? Explain.
4. Using $\vec{F}\Delta t = m\Delta\vec{v}$, which cart had the greater force?

## Applications

1. Explain why a target shooter might prefer to shoot a more massive gun.



## Law of Conservation of Momentum

The **law of conservation of momentum** states: *The momentum of any closed, isolated system does not change.* It doesn't matter how many objects are in the system. It is only necessary that no objects enter or leave the system and that there is no net external force on the system. Because all interactions can be classified as collisions in one form or another, the law of conservation of momentum is a very powerful tool.

As an example, imagine two freight cars, A and B, each with a mass of $3.0 \times 10^5$ kg, as seen in Figure 9–6. Car B is moving at +2.2 m/s while car A is at rest. The system is composed of the cars A and B. Assume that the cars roll without friction so there is no net external force. Thus, the two cars make up a closed, isolated system, and the momentum of the system is conserved. When the two cars collide, they couple and move away together. We can use conservation of momentum to find the velocity of the coupled cars.

The masses of the cars are equal, so we can write $m_A = m_B = m$. The initial velocity of A is zero, so $\vec{v}_A = 0$ and $\vec{p}_A = 0$. The initial velocity of B is $\vec{v}_B$. The initial momentum of B is $m\vec{v}_B$ or $\vec{p}_B$. Thus, the initial momentum of the system is

$$\vec{p}_A + \vec{p}_B = \vec{p}_B = m\vec{v}_B.$$

After the collision, the two coupled cars of course have the same velocity, $\vec{v}'_A = \vec{v}'_B = \vec{v}'$. Because their masses are also equal, $\vec{p}'_A = \vec{p}'_B = m\vec{v}'$. Therefore, the final momentum of the system is

$$\vec{p}'_A + \vec{p}'_B = 2m\vec{v}'.$$

By the law of conservation of momentum,

$$\vec{p}'_A + \vec{p}'_B = \vec{p}'_A + \vec{p}'_B$$

$$m\vec{v}_B = 2m\vec{v}'$$

$$\vec{v}_B = 2\vec{v}'$$

$$\text{or } \vec{v}' = \tfrac{1}{2}\vec{v}_B.$$

Since $\vec{v}_B = +2.2$ m/s, $\vec{v}' = \tfrac{1}{2}(+2.2 \text{ m/s}) = +1.1$ m/s.

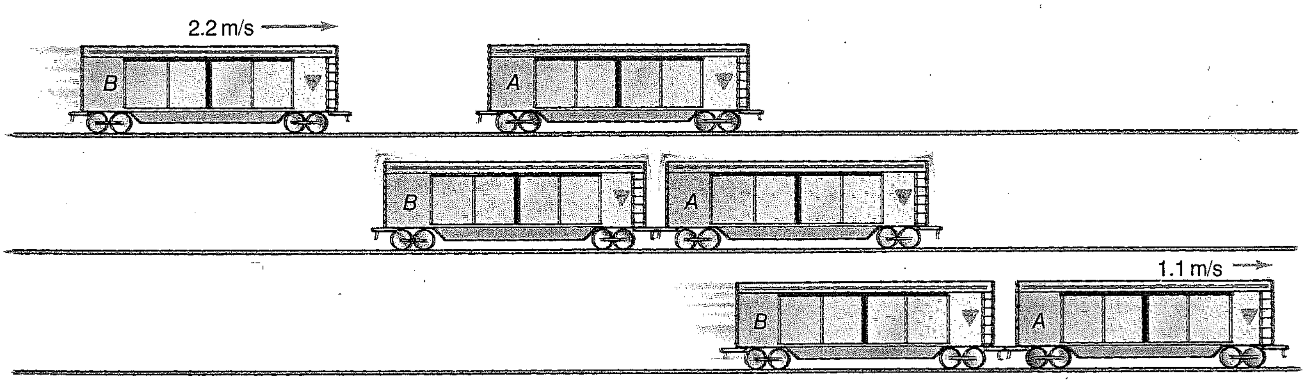
After the collision, the two cars move together with half the velocity of the moving car B before the collision.

# F.Y.I.

It takes a fully loaded supertanker, travelling at a normal speed of 16 kn, at least 20 min to stop. If an immobile object appeared suddenly as much as 5 km away, a collision would occur.

The total momentum of an isolated system always remains constant.

**FIGURE 9–6. The total momentum of the freight car system after collision is the same as the total momentum of the system before collision.**



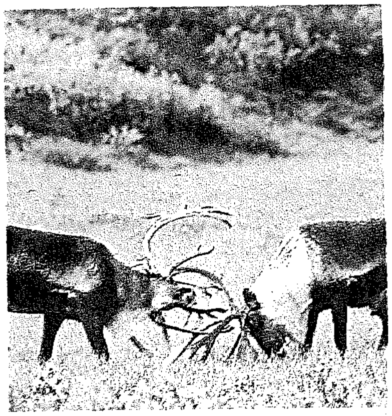


**FIGURE 9–7. The horns of two elk fighting often become entangled, producing an inelastic collision.**

The momentum of the two cars before the collision is given by

$$\begin{aligned}\vec{p}_A + \vec{p}_B &= m_A\vec{v}_A + m_B\vec{v}_B \\ &= (3.0 \times 10^5 \text{ kg})(0 \text{ m/s}) + (3.0 \times 10^5 \text{ kg})(+2.2 \text{ m/s}) \\ &= +6.6 \times 10^5 \text{ kg·m/s}.\end{aligned}$$

The final momentum of the system is given by

$$\begin{aligned}\vec{p}'_A + \vec{p}'_B &= m_A\vec{v}'_A + m_B\vec{v}'_B \\ &= (3.0 \times 10^5 \text{ kg})(+1.1 \text{ m/s}) + (3.0 \times 10^5 \text{ kg})(+1.1 \text{ m/s}) \\ &= +6.6 \times 10^5 \text{ kg·m/s in the direction of car B's original motion.}\end{aligned}$$

We can also look at the changes in momentum of individual parts of the system. The change in momentum of car A is

$$\begin{aligned}\Delta\vec{p}_A &= \vec{p}'_A - \vec{p}_A \\ &= m_A\vec{v}'_A - m_A\vec{v}_A \\ &= (3.0 \times 10^5 \text{ kg})(1.1 \text{ m/s}) - (3.0 \times 10^5 \text{ kg})(0 \text{ m/s}) \\ &= +3.3 \times 10^5 \text{ kg·m/s}.\end{aligned}$$

The change in momentum of car B is

$$\begin{aligned}\Delta\vec{p}_B &= \vec{p}'_B - \vec{p}_B \\ &= m_B\vec{v}'_B - m_B\vec{v}_B \\ &= (3.0 \times 10^5 \text{ kg})(1.1 \text{ m/s}) - (3.0 \times 10^5 \text{ kg})(+2.2 \text{ m/s}) \\ &= -3.3 \times 10^5 \text{ kg·m/s}.\end{aligned}$$

The magnitude of the momentum lost by car B ($-3.3 \times 10^5$ kg·m/s) is equal to the magnitude of the momentum gained by car A ($+3.3 \times 10^5$ kg·m/s).

The freight cars illustrate two important features of any collision. First, for the closed, isolated system as a whole, the total momentum is the same before and after the collision.

$$\vec{p}_A + \vec{p}_B = \vec{p}'_A + \vec{p}'_B$$

Second, the momentum gained by one part of the system is lost by the other part. Momentum is transferred,

$$\Delta\vec{p}_A = -\Delta\vec{p}_B.$$

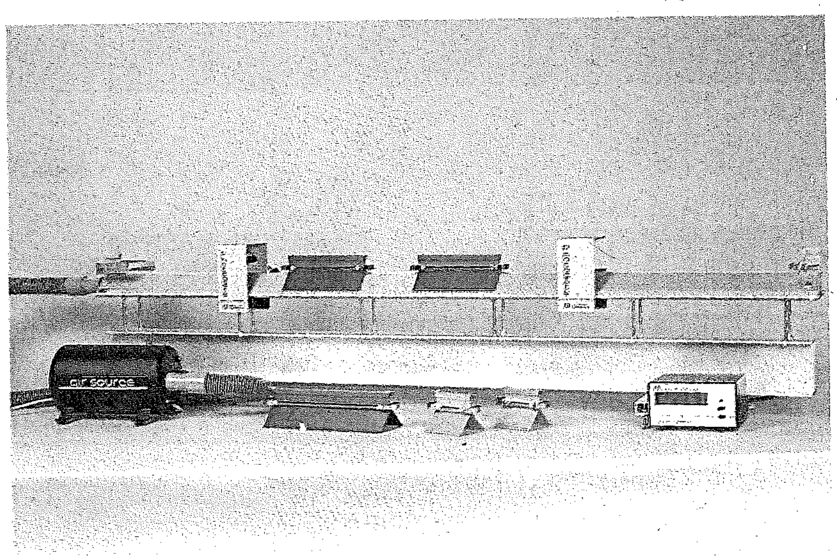



**FIGURE 9–8. Air tracks provide nearly frictionless surfaces to simplify the measurement of the change in momentum during collisions.**

## Example Problem

### Conservation of Momentum—1

Glider A of mass 0.355 kg moves along a frictionless air track with a velocity of 0.095 m/s. It collides with glider B of mass 0.710 kg moving in the same direction at a speed of 0.045 m/s. After the collision, glider A continues in the same direction with a velocity of 0.035 m/s. What is the velocity of glider B after the collision?

**Given:** $m_A = 0.355\ \text{kg}$

$\vec{v}_A = +0.095\ \text{m/s}$

$m_B = 0.710\ \text{kg}$

$\vec{v}_B = +0.045\ \text{m/s}$

$\vec{v}'_A = +0.035\ \text{m/s}$

**Unknown:** $\vec{v}'_B$

**Basic equation:** $\vec{p}_A + \vec{p}_B = \vec{p}'_A + \vec{p}'_B$

**Solution:** $\vec{p}_A + \vec{p}_B = \vec{p}'_A + \vec{p}'_B$, so

$\vec{p}'_B = \vec{p}_B + \vec{p}_A - \vec{p}'_A$

$m_B\vec{v}'_B = m_B\vec{v}_B + m_A\vec{v}_A - m_A\vec{v}'_A$

or

$$\vec{v}'_B = \frac{m_B\vec{v}_B + m_A\vec{v}_A - m_A\vec{v}'_A}{m_B}$$

$$= \frac{(0.710\ \text{kg})(+0.045\ \text{m/s}) + (0.355\ \text{kg})(+0.095\ \text{m/s}) - (0.355\ \text{kg})(+0.035\ \text{m/s})}{0.710\ \text{kg}}$$

$$= +0.075\ \text{m/s}$$

## Practice Problems

**5.** A 0.105-kg hockey puck moving at 48 m/s is caught by a 75-kg goalie at rest. With what speed does the goalie slide on the ice?

**6.** A 35.0-g bullet strikes a 5.0-kg stationary wooden block and embeds itself in the block. The block and bullet fly off together at 8.6 m/s. What was the original velocity of the bullet?

**7.** A 35.0-g bullet moving at 475 m/s strikes a 2.5-kg wooden block. The bullet passes through the block, leaving at 275 m/s. The block was at rest when it was hit. How fast is it moving when the bullet leaves?

▶ **8.** A 0.50-kg ball traveling at 6.0 m/s collides head-on with a 1.00-kg ball moving in the opposite direction at a velocity of −12.0 m/s. The 0.50-kg ball moves away at −14 m/s after the collision. Find the velocity of the second ball.

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### Using Your CALCULATOR

Using the memory function of your calculator can simplify the evaluation of complex expressions. Note that, since the third term in the numerator is subtracted, the +/− key is used to make the term negative before the subtotal is recalled from memory and added.

$$v'_B = \frac{m_Bv_B + m_Av_A - m_Av'_A}{m_B}$$

| Keys | Display |
|---|---|
| [.710] [×] [.045] [=] [M_in] | 0. 03195 |
| [AC] [.355] [×] [.095] [=] | 0. 033725 |
| [+] [MR] [=] [M_in] [AC] | 0. 065675 |
| [.355] [×] [.035] [=] [+/−] | −0. 012425 |
| [+] [MR] [=] | 0. 05325 |
| [÷] [.710] [=] | 0. 075 |

$v'_B = 0.075\ \text{m/s}$

The momentum gained by one object in an interaction is equal to the momentum lost by the other object.

## Internal and External Forces

An internal force cannot change the total momentum of a system.

In the example of the collision of two railroad cars, the cars exerted forces on each other. The forces between objects within a system are called **internal forces.** The total momentum of a system is conserved, or stays the same, only when it is closed and isolated from external forces. What if we define the system as car A only? When car A collides with car B, a force is exerted by an object outside the system. An **external force** acts. Our system is no longer isolated, and the momentum of the system is not conserved. You see, it is very important to define the system carefully.

It is important to define a system carefully. A force may be either internal or external depending on the system definition.

Consider the two roller bladers shown in Figure 9–9 as an isolated, closed system. That is, we will assume no external forces because the surface is so smooth that there is very little frictional force. Blader A has a mass of 60.0 kg; blader B has a mass of 45.0 kg. At first the bladers are standing still. The initial momentum of the system is zero. Blader A gives blader B a push. Now both bladers are moving. Only an internal force has been exerted, so the momentum of the system is conserved. We can find the relative velocities of the two bladers using conservation of momentum.

$$\vec{p}_A + \vec{p}_B = \vec{p}'_A + \vec{p}'_B$$

$$0 = \vec{p}'_A + \vec{p}'_B$$

or

$$\vec{p}'_A = -\vec{p}'_B$$

$$m_A\vec{v}'_A = -m_B\vec{v}'_B$$

Since the initial momentum of the system is zero, the vector sum of the momenta after the collision must also be zero. Therefore, the mo-

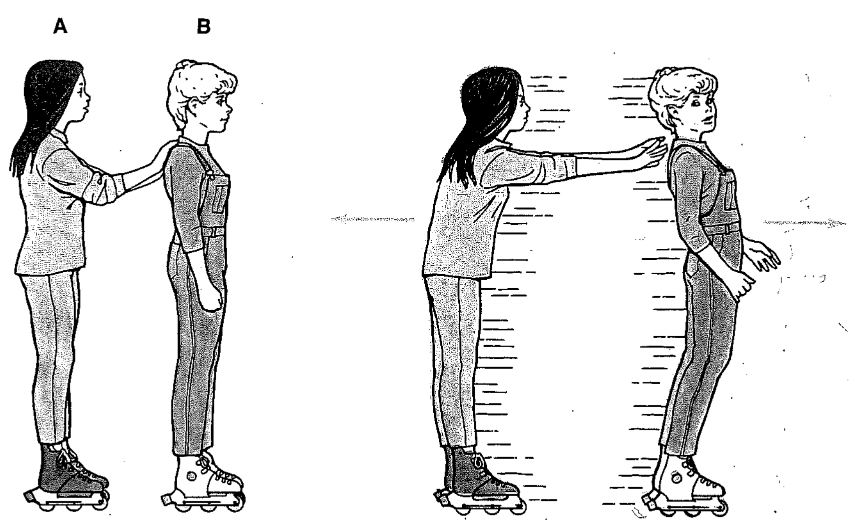



**FIGURE 9–9. The internal forces exerted by these bladers cannot change the total momentum of the system.**

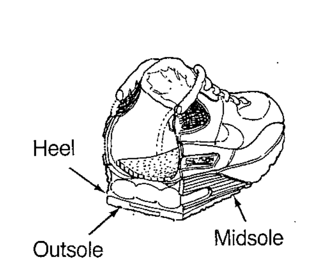
# Physics and technology

## RUNNING SHOE DESIGN

About $3 million a year are spent on athletic shoes in North America. Not only are athletic shoes big business, they are also "high-tech". Athletic shoes are now engineered to give the best performance for a specific use. Because running shoes take the most beating during use, they are the testing ground for other kinds of athletic shoes.

Each time an average runner's foot strikes the ground, the runner's body must absorb the force of two to four times its own weight. A goal of athletic shoe engineering is to reduce the stress of this load on the leg and foot. The vertical component of momentum goes to zero when the runner's foot hits the ground, requiring a large impulse. By using materials that lengthen the time of impact on the runner's foot, shoe engineers can reduce the force on the foot.

Therefore, the midsole, sandwiched between the insole and the outsole, becomes the most important part of the shoe. The midsole, which cushions the foot and absorbs part of the force on the runner's body, can be made of a wide variety of materials. One manufacturer uses a three-dimensional, woven-coil fabric developed by NASA for moonboots. Another uses an encapsulated silicone-based gel. Some midsoles consist of polyurethane that con-




tains trapped bubbles of air or a freon-type gas. A recent design provides the runner with two sets of air cylinders that can be placed beneath the insole.

• What is the value of a design feature such as this?

menta of the two bladers are equal in magnitude and opposite in direction. Their velocities, however, are not equal. The more massive blader moves more slowly than the smaller one.

The system we defined included both bladers. Therefore, when one blader pushed on the other, the force was an internal force. The total momentum of the system remained the same. On the other hand, if we had defined the system as only blader B, there would have been an external force, and the total momentum of the system would have changed.

A system can contain more than two objects. For example, a stoppered flask filled with gas, Figure 9–10, is a system consisting of many particles. The gas particles are constantly colliding with each other and the walls of the flask. Their momenta are changing with every collision. In these collisions, the momentum gained by one particle is equal to the momentum lost by the other particle. Thus, the total momentum of the system does not change. The total momentum of any closed, isolated system is constant.

How does a rocket accelerate in space? This is another example of conservation of momentum. Fuel and oxidizer chemically combine, and the hot gases that result are expelled from the rocket's exhaust nozzle at high speed. Before combustion, the rocket with its unburned fuel

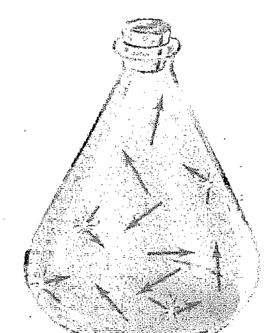


**FIGURE 9–10. The total momentum of an isolated system is constant.**

moves forward at some constant speed. It has a momentum $mv$. After the firing, the mass of the combustion gases moves backward with high relative velocity. In order to conserve momentum, the rocket, with its mass reduced by the burned fuel, must move with increased speed in the forward direction.

**FIGURE 9–11. The Space Shuttle and its exhaust gases represent an isolated system in which momentum is conserved.**

## Example Problem

### Conservation of Momentum—2

An astronaut at rest in space with mass 84 kg fires a thruster that expels 35 g of hot gas at 875 m/s. What is the velocity of the astronaut after firing the shot?

**Given:** before firing: $m_A = 84\text{ kg}$

$\vec{v}_A = 0\text{ m/s}$

$m_B = 0.035\text{ kg}$

$\vec{v}_B = 0\text{ m/s}$

after firing: $\vec{v}'_B = 875\text{ m/s}$

**Unknown:** $\vec{v}'_A$

**Basic equation:** $\vec{p}_A + \vec{p}_B = \vec{p}'_A + \vec{p}'_B$

**Solution:** $\vec{p}_A + \vec{p}_B = \vec{p}'_A + \vec{p}'_B$

But $\vec{v}_A = \vec{v}_B = 0$, so $\vec{p}_A = \vec{p}_B = 0$, and $\vec{p}'_A = -\vec{p}'_B$

$$m_A\vec{v}'_A = -m_B\vec{v}'_B \quad \vec{v}'_A = -\frac{m_B\vec{v}'_B}{m_A}$$

$$\vec{v}'_A = -\frac{(0.035\text{ kg})(+875\text{ m/s})}{84\text{ kg}} = -0.36\text{ m/s}$$

The astronaut recoils in a direction opposite that of the moving gas.

## Practice Problems

9. A 4.00-kg model rocket is launched, shooting 50.0 g of burned fuel from its exhaust at an average velocity of 625 m/s. What is the velocity of the rocket after the fuel has burned? (Ignore effects of gravity and air resistance.)
10. A thread holds two carts together on a frictionless surface as in Figure 9–12. A compressed spring acts upon the carts. After the thread is burned, the 1.5-kg cart moves with a velocity of 27 cm/s to the left. What is the velocity of the 4.5-kg cart?

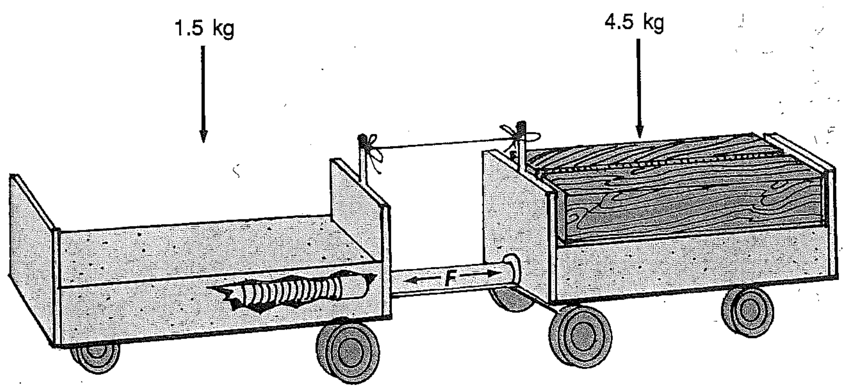



**FIGURE 9–12. Use with Practice Problem 10.**

11. Two campers dock a canoe. One camper steps onto the dock. This camper has a mass of 80.0 kg and moves forward at 4.0 m/s. With what speed and direction do the canoe and the other camper move if their combined mass is 110 kg?

▶ 12. A gunner sets up his 225-kg cannon at the edge of the flat top of a high tower. It shoots a 4.5-kg cannon ball horizontally. The ball hits the ground 215 m from the base of the tower. The cannon also moves, on frictionless wheels, and falls off the back of the tower, landing on the ground.

  a. What is the horizontal distance of the cannon's landing, measured from the base of the back of the tower?

  b. Why do you not need to know the width of the tower?

## Conservation of Momentum in Two Dimensions

The total momentum of a system is the vector sum of the momenta of all the parts of the system.

Until now we have looked at momentum in one dimension only. The law of conservation of momentum, however, holds for all isolated, closed systems. It is true regardless of the directions of the particles before and after they collide.

Figure 9–13a shows the result of billiard ball A striking stationary ball B. The momentum of the moving ball is represented by the vector $\vec{p}_A$. The momentum of the ball at rest is zero. Therefore, the total momentum of the system is the vector $\vec{p}_A$ going toward the right. After the collision, the momenta of the two balls are represented by the vectors $\vec{p}'_A$ and $\vec{p}'_B$. Figure 9–13b shows that the vector sum of $\vec{p}'_A$ and $\vec{p}'_B$ equals the original momentum, $\vec{p}_A$. The vertical and horizontal components of the vectors can also be added. The initial momentum has no vertical or $y$ component, so the vector sum of the final vertical components of the two balls, $\vec{p}'_{Ay}$ and $\vec{p}'_{By}$, must be zero. They are equal in magnitude but opposite in direction. The sum of the final horizontal components, $\vec{p}'_{Ax}$ and $\vec{p}'_{Bx}$, must equal the original momentum, which is totally horizontal.

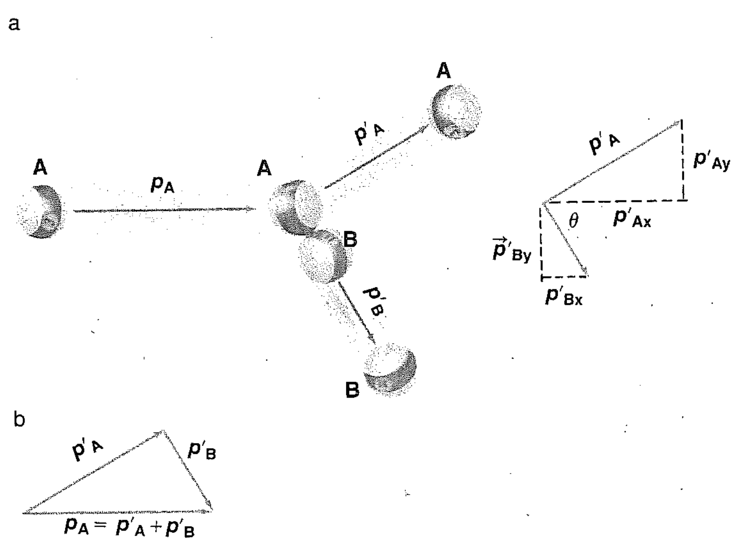



**FIGURE 9–13. The law of conservation of momentum holds for all isolated, closed systems, regardless of the directions of objects before and after they collide (a). The vector sum of the momenta is constant (b).**

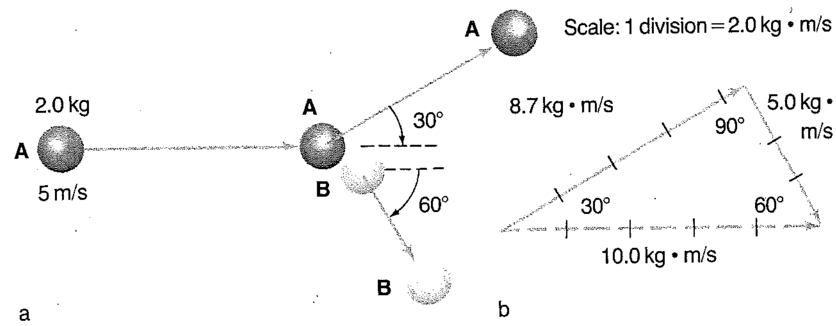



**FIGURE 9–14. Use with the following Example Problem.**

# Example Problem

## Conservation of Momentum in Two Dimensions

A 2.00-kg ball, A, is moving at a velocity of 5.00 m/s. It collides with a stationary ball, B, also of mass 2.00 kg, Figure 9–14. After the collision, ball A moves off in a direction 30.0° to the left of its original direction. Ball B moves off in a direction 90.0° to the right of ball A's final direction. **a.** Draw a vector diagram to find the momentum of ball A and of ball B after the collision. **b.** Find the velocities of the balls after the collision.

**Given:** $m_A = 2.00\text{ kg}$

$\vec{v}_A = +5.00\text{ m/s}$

$m_B = 2.00\text{ kg}$

$\vec{v}_B = 0.00\text{ m/s}$

**Unknowns:** $\vec{p}_A, \vec{p}'_A, \vec{p}'_B, \vec{v}'_A, \vec{v}'_B$

**Basic equation:** $\vec{p} = \vec{p}'$, so $\vec{p}_A + \vec{p}_B = \vec{p}'_A + \vec{p}'_B$

**Solution:** Find the initial momenta.

$\vec{p}_A = m_A\vec{v}_A = +10.0\text{ kg}\cdot\text{m/s}$ $\quad \vec{p}_B = 0$

Therefore, $\vec{p} = \vec{p}_A + \vec{p}_B = +10.0\text{ kg}\cdot\text{m/s}$

The vector representing the initial momentum is an arrow to the right. The initial and final momenta are equal, $\vec{p} = \vec{p}'$, so $\vec{p}_A + \vec{p}_B = \vec{p}'_A + \vec{p}'_B$. Because the angle between the final directions of ball A and ball B is 90°, the vector triangle is a right triangle with the initial momentum as the hypotenuse. The vector $\vec{p}'_A$ is at 30.0°. We can use trigonometry to find the magnitudes of the vectors.

Ball A

$$\cos 30° = \frac{\vec{p}'_A}{\vec{p}'}$$

$$\vec{p}'_A = \vec{p}' \cos 30° = (1.0 \times 10^1\text{ kg}\cdot\text{m/s})(0.87) = 8.7\text{ kg}\cdot\text{m.s}$$

Ball B

$$\sin 30° = \frac{\vec{p}'_B}{\vec{p}'}$$

$$\vec{p}_B = \vec{p}' \sin 30° = (1.0 \times 10^1\text{ kg}\cdot\text{m/s})(0.50) = 5.0\text{ kg}\cdot\text{m/s}$$

Now use the definition of momentum, $\vec{p} = m\vec{v}$, to find the magnitudes of the velocities.

$$\vec{p}'_A = m_A\vec{v}'_A \qquad \vec{p}'_B = m_B\vec{v}'_B$$

$$\vec{v}'_A = \frac{\vec{p}'_A}{m_A} \qquad \vec{v}'_B = \frac{\vec{p}'_B}{m_B}$$

$$= \frac{8.7\ \text{kg}\cdot\text{m/s}}{2.0\ \text{kg}} = 4.4\ \text{m/s} \qquad = \frac{5.0\ \text{kg}\cdot\text{m/s}}{2.0\ \text{kg}} = 2.5\ \text{m/s}.$$

## Practice Problems

**13.** A 1325-kg car moving north at 27.0 m/s collides with a 2165-kg car moving east at 17.0 m/s. They stick together. Draw a vector diagram of the collision. In what direction and with what speed do they move after the collision?

**14.** A 6.0-kg object, A, moving at velocity 3.0 m/s, collides with a 6.0-kg object, B, at rest. After the collision, A moves off in a direction 40.0° to the left of its original direction. B moves off in a direction 50.0° to the right of A's original direction.

**a.** Draw a vector diagram and determine the momenta of object A and object B after the collision.

**b.** What is the velocity of each object after the collision?

▶**15.** A stationary billiard ball, mass 0.17 kg, is struck by an identical ball moving at 4.0 m/s. After the collision, the second ball moves off at 60° to the left of its original direction. The stationary ball moves off at 30° to the right of the second ball's original direction. What is the velocity of each ball after the collision?

## CONCEPT REVIEW

**2.1** Two soccer players come from opposite directions. They leap in the air to try to hit the ball, but collide with each other instead, coming to rest in midair. What can you say about their original momenta?

**2.2** During a tennis serve, momentum gained by the ball is lost by the racket. If momentum is conserved, why doesn't the racket's speed change much?

**2.3** Someone throws a heavy ball to you when you are standing on a skateboard. You catch it and roll backward with the skateboard. If you were standing on the ground, however, you would be able to avoid moving. Explain both using momentum conservation.

**2.4** A pole vaulter runs toward the launch point with horizontal momentum. Where does the vertical momentum come from as the athlete vaults over the crossbar?

**2.5** Critical Thinking: A hockey player deflects a fast-moving puck through an angle of 90°. The puck's speed is not changed. By considering the two components of the impulse given the puck by the stick, tell in what direction the stick must have been moving when it deflected the puck.

**FIGURE 9–15. An inelastic collision.**